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# The Mathematics Teacher

OCTOBER 1954

*The role of insight in the learning of mathematics*
HOWARD F. FEHR

*Improving the learning of mathematics*
KENNETH P. KIDD

*The function concept in secondary school mathematics*
J. HOUSTON BANKS

*The official journal of*
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# *The role of insight in the learning of mathematics*

HOWARD F. FEHR, *Teachers College, Columbia University, New York.*

*Insightful learning does not come about by telling the students how to solve problems. It comes about by making use of the heuristic method—giving hints and clues, examining solutions, and approaching a problem in many ways.*

THE QUESTIONS of problem-solving and the nature of trial and error, and the insight in arriving at meaningful solutions to thought problems, have been investigated by a rather large number of psychologists. These researchers have been interested in the scientific aspects of the process, in finding out what it is that goes on in the mind or the organism as they solve problems. They are busy with the logical structure and explanation of thought, analysis, insight, etc. They have not, however, been concerned with how to initiate or to direct mental processes that lead to successful solutions. This latter is the task of the educator and the teacher, namely, to apply to the teaching process the findings of psychologists so that they may develop a greater problem-solving ability on the part of their students.

There is no common agreement on what insight is or how insightful learning is brought about. To most psychologists the aspect of "suddenness" of the solution is the key feature of insight. All of us can recollect how, after searching in vain for solutions, "all of a sudden"—like a flash—the problem resolves itself: insight has occurred. At times, one sees a situation in which the action to the solution is evident and requires no thinking. The insight is present immediately, perhaps because of an earlier experience in which the solution or necessary action to the solution was derived. To illustrate this phase of learning, consider the following problem[1] (Figure 1): "$C$ is a point on $AB$, with $AC>CB$. With $C$ as center, quadrants $BE$ and $AD$ are described on the same side of $AB$. $D$ and $E$ are joined by a semi-circle forming a cusp at $D$ and a flex point at $E$. Show the area enclosed by the figure $ABEDA$ is equal to that of the semi-circle on $AB$."

Here your past experience gives the insight at once. Find the area of the semicircle with diameter $a+b$. Then find the sum of the areas of quadrant $BE$ and $AD$ and diminish it by semicircle $DE$. The two

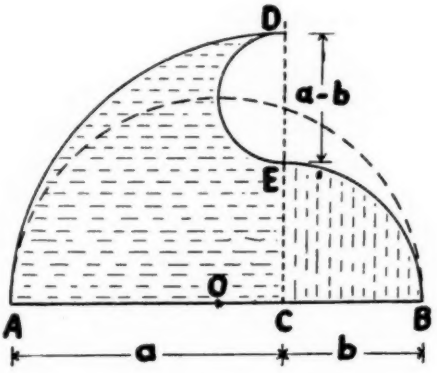



*Figure* 1

[1] Leon Bankoff, "Problem No. 2377," *School Science and Mathematics*, LIII (November 1953), 665.

areas are identical. Further investigation shows that this solution is the common algebraic form

$$(a+b)^2=2a^2+2b^2-(a-b)^2 \qquad \text{(A)}$$

This concept of form, or configuration, or gestalt, was introduced because it is one of the essential features of insightful learning. To introduce other characteristics, it will be necessary to clarify the concept of problem-solving as it is used in this paper.

A problem arises only when you know a goal but do not know how you can attain the goal. The goal may be specific or general. If, in a given situation, the action which is to be taken to reach a goal is evident, as in the foregoing example, then there is no problem, as insight has already been attained. If, however, it is impossible to go from a given situation to a sought-for goal by direct action, and if there is a felt need on the part of the individual involved to get to the goal, thinking must take place. The thinking must devise those actions or operations (mental or physical) that will get the individual to his desired goal.

Suppose we extend the foregoing problem to permit the point $C$ to divide $AB$ externally so that $AC>CB$ (see Figure 2). How, then, will we draw the figure, and will the theorem still be valid? If you have not previously attacked this situation, and you are interested, it will pose a problem. Note in the original theorem that arc $BE$ and arc $DE$ were drawn counterclockwise. We maintain this in Figure 2. Then the theorem is also true if we consider the area II as negative. To arrive at this goal note that the area of quadrant $ACD$ is $\pi a^2/4$, the area of quadrant $BCE$ is $\pi b^2/4$, and the area of semicircle $DE$ is $\pi[(a+b)^2/8]$. We remove this semicircle from the two quadrants by considering the area $MBE$ as still to be removed. Hence, the area of the semicircle on $AB$, which is $\pi[(a-b)^2/8]$ is equal to area I minus area II (or if area II is negative, we can say area I plus area II). Note that the algebraic form in this extension is

$$(a-b)^2=2a^2+2b^2-(a+b)^2 \qquad \text{(B)}$$

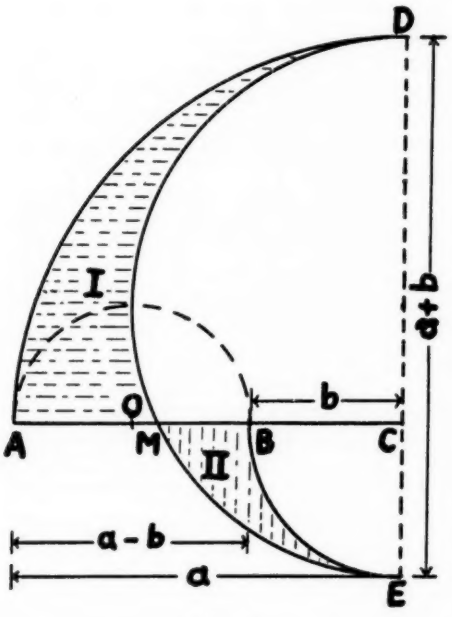



*Figure* 2

There is no doubt that such an extension as this gives greater insight into the nature of the theorem. There is thus involved in insight an element of *generalization by extension*. There is still a further element that aids in insightful learning. In the examples, is it possible to make the extension *conform* in general structure to the original problem? It is if we consider $AC$ as positive and $CB$ as negative. Line $AB$ then becomes $(a+b)$ and $DE$ becomes $(a-b)$. Under this *reconstruction* of the configuration, we have a more inclusive generalization and one that gives more insight into the mathematical relations involved. The algebraic form (B) is identical to form (A) if $b$ is negative, and (A) can be applied universally to the configuration. In learning mathematics by the use of insight we continuously make use of *reconstructions*. The theory of exponents in algebra will

suggest itself as an example similar to the geometric illustration just given.

Problem-solving is explained commonly by two theories, the one called "Trial and Error," the other "Analysis and Insight." Woodworth and others have described trial-and-error learning as:

1. A set to reach a goal
2. Inability to see any way clear to the goal
3. Exploring the situation
4. Seeing or somehow finding leads, possible ways to reach the goal
5. Trying these leads, backing off when blocked in one lead, and trying another
6. Finally finding a good lead and reaching the goal
7. Practicing the clear path until it is well established. The more practice the longer the retention.

All of us have experienced this type of learning in solving verbal problems in algebra and geometry originals, and on through higher mathematics. In fact, this is the way many of us were taught to solve problems, and today many of our pupils use this method, albeit with not too much success. However, we must grant that in all our solutions to problems there is an element of trial and error as we grope for possible clues to the answer.

Insightful solution is characterized by the following traits:

1. Surveying the problem situation as a whole
2. Perceiving relationships between, and properties of, the elements to the whole situation and the whole situation to the elements
3. Organizing and reorganizing the elements into patterns or gestalts
4. Emergence of a complete solution from the structure of the situation
5. Sudden discovery and high retention of the learning.

In trial-and-error learning, when one finally arrives at a solution, he frequently does not see the path or all the steps from the beginning to the end. It is a chain of elements that must be gone over and over to give complete understanding. In insightful learning, isolated sequential connections do not occur. The solution is a configuration of elements related to the whole situation rather than merely to its adjacent elements. It is a pattern. In attacking a problem by insightful methods, first an analysis is made of the situation (or field) to gain understanding and to permit appropriate transformations. The attack on the problem is always from "*above*"; that is, on the situation as a whole and not from "below"; that is, with one element and its relation to an adjacent element. This insightful approach aids in avoiding circular paths.

Trial-and-error reasoning frequently leads back to an element already established. How many times has the reader or his students, by trial and error, arrived back at the very statement that began the argument rather than at the desired conclusion, both in geometry and in algebra? However, insight is not a magical revelation of the solution. The reorganization of previous learning must be directed in some manner, and here cues, hints, and guidance take on the same aspect as trying a new lead. The distinction lies in the way the cue or hint is applied, the one through starting a new chain (trial and error), the other in completing a new configuration or structure.

To illustrate these aspects consider the following problem[2] (see Figure 3).

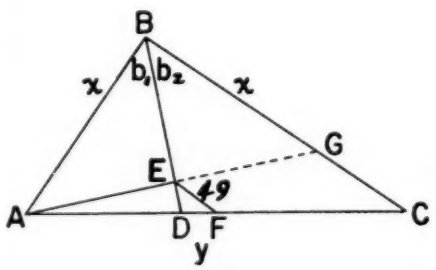



*Figure 3*

[2] Leon Bankoff, "Problem No. 180," *Mathematics Magazine*, XXVII (September–October 1953), 52.

In right triangle $ABC$, whose sides are integers $AC>BC>AB$, the bisector of angle $ABC$ meets $AC$ at $D$. $E$ is the projection of $A$ upon $BD$, and $F$ is the midpoint of $AC$. If $EF=49$, find $AB$, $BC$, and $AC$. In attempting the solution by a trial-and-error procedure, one recalls angle bisector theorems ($AD:DC=AB:BC$), the Pythagorean theorem ($AC^2=CB^2+BA^2$), and the median theorem ($BF=AF=FC=\frac{1}{2}AC$). None of these approaches from "below" gives a path to the solution. They do not involve the essential element in the problem, namely, the relationship of $EF$ to the total situation. Further, the realization that any one of these relations does not work merely leads to the use of another. The *realization of why* a particular relation does not work, why it does not fit into the desired construction, is far more important in gaining a key to the solution.

A total configurational approach would analyze the given relations with respect to the entire figure. The questions: how is the bisector of angle $B$ related to $AE$?; what reorganization or reconstruction does this suggest?; and how is $EF$ related to the figure? (it must be related to the sides to achieve our goal). All these questions suggest completing the figure by extending $AE$ to $G$. Now insightful relations appear. They literally jump out of the figure. [$EF=\frac{1}{2}GC$; $BG=AB$; $x^2+(x+98)^2=y^2$. We want integral solutions; Diophantine equation, etc.] According to Duncker, it is this functional aspect of a solution, the relation of the *essential element* to the whole situation, that mediates the understanding of a solution. Without this functional understanding, we get errors, or deficient solutions, and the transfer of the learning to a new situation is highly improbable.

We thus see that finding a solution to a problem is done through a type of thinking that consists in combining parts of previous experience with elements of a given situation into new patterns until insight or a total configurational relationship occurs. The combination of these elements into a new pattern is always under *"direction,"* and the direction is best determined by the functional elements in the problem. Direction is obtained by continually reformulating the problem out of its original setting. The training for this type of solution involves the ability to use (1) recall of past solutions, (2) key elements, (3) abstractions, (4) generalizations, (5) analyses, and (6) reconstructions or new patterns of all the elements in the situation.

In a study of methods of solution, the following problem by Bankoff was submitted to a group of teachers with instructions to do all their thinking aloud as they proposed procedures[3] (see Figure 4).

$P$ is any point on a semicircle described externally on side $BC$ of square $BDEC$. $PD$ and $PE$ cut $BC$ in $F$ and $G$ respectively. Show that $BF\cdot GC=FG^2$.

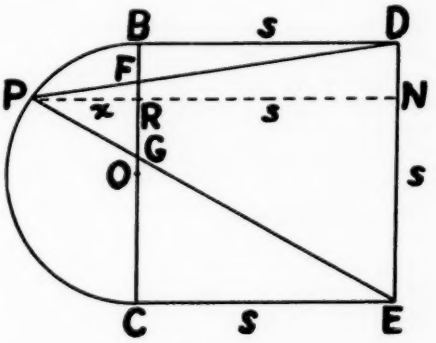



*Figure 4*

The first person suggested we draw $PB$ and $PC$. Why? It would form a right triangle, and some relation might be found. (Obviously this did not lead to a solution.)

A second person suggested dropping a perpendicular from $P$ to $BC$. Why? It would be a mean proportional between the segments of $BC$. Maybe we could get other relations. What? A block was reached.

A third person suggested $\triangle PFG$ was similar to $\triangle PDE$. What are we to do with it? $FG$ is in same ratio to $DE$ as corresponding sides. The proportion was written. No further steps were seen.

[3] Leon Bankoff, "Problem No. 2379," *School Science and Mathematics*, LIII (November 1953), 665.

A fourth person suggested continuing the perpendicular from $P$ to side $DE$. Why? Get the ratio of the altitudes of the triangles to $FG$ and $DE$. This was written down and again a block was reached. All of these approaches are indicative of trial-and-error solutions, not insightful approaches.

In order to illustrate an insightful approach the following hints were given.

1. Do you believe that the theorem is true? Have you varied the position of $P$ and studied the relation of $BF$, $FG$, and $GC$? A little experimentation showed the variation in these parts and revealed the following: While $PR(x)$ varied, $RN$ remained the same, equal to the side of the square. Hence, $FG$ is related to $x$ and $s$. Where are all the $s$ elements in the figure? How will they be related to the solution?

2. How are the parts of $BC$ related to $x$ and $s$ in the figure? This brought forth a number of relations. (One of which led to circular reasoning, namely, the relation of $FR$ to $DN$ and $RG$ to $NE$, which brought this student right back to the relation of $FG$ to $DE$.)

3. The relations

$$(FG)^2=\frac{x^2s^2}{(s+x)^2}\,; \qquad \frac{BF}{FR}=\frac{s}{x}\,; \qquad \frac{CG}{GR}=\frac{s}{x}$$

were accepted as functional to the solution. The only missing link was the relation of $FR$ and $GR$. One student suggested the elimination of $FR$ and $GR$. Another suggested using the addition property on the last two equations to obtain $s+x$ in each denominator. (He had reformulated the initial problem into obtaining an identity of equations.)

4. Writing

$$\frac{BF}{BR}=\frac{s}{s+x} \text{ and } \frac{CG}{CR}=\frac{s}{s+x}$$

produced immediately

$$BF\cdot CG=BR\cdot CR\cdot\frac{s^2}{(s+x)^2}\,.$$

A glance at the figure and insight (total configurational relation) came suddenly to practically all members of the class. By watching the faces of each individual in the class, the instructor could clearly determine the moment insight occurred. All at once, in the mind of a student, the pieces fitted together into the pattern he could interpret and that gave him his goal.

5. What shall we do now? This was a puzzling question. A few students suggested going over the proof. This was rejected by most of the other students, who said they saw the relationships in their entirety as the proof evolved. The instructor then proposed they extend the relationship, if possible, considering the relation when a circle instead of a semicircle is given. Figure 5 shows that the relation still held, and in the solution $s-x$ replaced $s+x$.

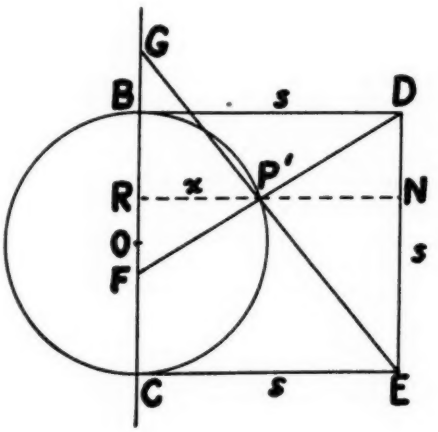



*Figure 5*

6. The students were asked to generalize both solutions into a single one. This brought forth the comment: Consider $x$ negative for the part of the circle interior to the square, then both solutions will be identical. Unable to get any further generalization, the instructor asked the class to consider the hyperbola with center at $O$ and equation $y^2-x^2=s^2/4$, with $BC$ the $y$-axis. This, for the first time brought forth the analytic approach to the problem, using $O$ as the origin and $BC$ as the

$y$-axis. The same relationship was found to hold in this case.

Thus the seizing of the functional element, the relationship of the elements in the problem to the whole problem, the reformulation and restructuring of the problem and the extension by generalization gave the insightful solution. The best approach was from "above," not from "below."

While these examples have shown how this type of solution is obtained, they have not shown how to teach so as to achieve these types of problem solutions. The rest of this paper is given over to some practical suggestions, all of which have appeared from time to time in the literature, but which few teachers have as yet put into effect in their teaching.

Insightful learning does not come about by telling students how to solve problems, and then having them reproduce the procedure on a given set of problems. Rather, it comes about by giving only hints and cues, guiding the selection of elements, and *having students examine their own solutions.* This calls for a *heuristic* approach to teaching. Especially is it necessary to have students select the functional elements of their solutions and discuss ways of detecting such elements. Many illustrations of this approach to problem-solving are presented in Wertheimer's *Productive Thinking*[4] and Duncker's *On Problem Solving.*[5] It is thus better to examine and compare several different solutions to the same problem than to solve three similar problems by the same method. The problem, "Find two numbers for which their sum is 7 and the sum of their squares is 25," can be solved by using:

(a) $x+y=7$ $\quad x^2+y^2=25$

(b) $x$ and $7-x$ $\quad x^2+(7-x)^2=25$

(c) $x^2+y^2+2xy=49$

$2xy=24$

$xy=12$

A comparison of these solutions gives a better picture of the total relations involved.

For genuine problem-solving, it is important that correct concepts be established rather than specific procedures. Consider the solution of a quadratic equation. Here almost always we tell how and expect students only to do the mechanics. The discovery of the solution should be as much a problem to the student as any distance-rate-time problem. What concept shall we develop? The concepts of "root," solution, and equation are fundamental. First we refer back to what the pupil has previously learned. *Insightful learning makes continuous use of past learning.* To solve $ax=b$, the pupil divided both sides of the equation by $a$. Now suppose he has to solve $x^2=25$. What shall he do? None of the procedures developed heretofore work. The pupil may try to divide each side of the equation by $x$, but this leads to a block. "What other operation do we have?" can be a cue. Then square root gives the answer $+5$. Perhaps a good student also suggests $-5$. Do these roots satisfy the equation? The functional elements are *roots* of an equation and *square roots of a number.* Here they are the same.

Next we might consider $x^2-4x=5$. Dividing by $x$ does not aid in the solution. What did we do before? Square root? But how do I take square root? Are there algebraic expressions for which you can take the square root? Can you recall such expressions? This leads to completing the square, $x^2-4x+4=5+4$. Now what shall we do? *Square root!* Refer back to $x^2=25$ to get a positive root and a negative root. Thus $x-2=3$ or $x-2=-3$. But insight

[4] Max Wertheimer, *Productive Thinking* (New York: Harper, 1945). See especially Chapters I, III, and V.

[5] Karl Duncker, "On Problem Solving," *Psychological Monographs,* Whole No. 270, American Psychological Association, Inc., Washington 16, D.C. See especially Chapter III (On Solution-Processes with Mathematical Problems) and Chapter VIII (On Functional Fixedness of Mathematical Solution-Elements).

comes not in getting $x=5$ or $x=-1$; it comes by relating the goal (solution) to the entire situation. That is, the square roots of $(x-2)^2$ are $+3$ and $-3$. The roots of the equation are 5 and $-1$. The further reference of these roots to the axis of the graph of the function $x^2-4x-5$ gives a larger configuration. It is only in the extending and generalizing of concepts and relationships that a meaningful structure of elementary algebra can be built. Finally, the pupil generalizes the solution for

$$ax^2+bx+c=0 \quad \text{to} \quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Here again, in most teaching, algebra ceases and arithmetic takes over as students are asked to use this formula to solve hundreds of quadratics. None of this busywork gives insight into the quadratic equation If it is skill in arithmetic we wish to develop, this is fine; but if it is understanding of the nature of algebra and of solutions of problems by the use of algebra, we are on the wrong road. It would be better learning to have pupils discover how to solve general quadratics by several other methods.

Real problem-solving involves seeing the problem as a whole, familiarity with all the elements of the problem situation, making analyses, seeking relations, getting a pattern of relationships, estimating and checking, and reorganization. We seldom solve real problems in organized deductive steps. This deductive structure is usually made after insight to the solution has been attained. For example, consider the following problem taken from the *Bulletin* of the Mathematical Association of America:[6]

" 'That was a good lunch; now for a good cigar and then I must catch the one o'clock train. Let's see—my watch says exactly nine o'clock but that can't be right. It's still running and well-wound. Now I remember I wound it and set it just this morning by the radio. Maybe I carelessly set the hands in reverse position. If so, exactly what time is it?' "

Are you interested in solving the problem? Do you have a set of rules, a step sequence that you are applying? An analysis of your solution to this problem, a check-up on how you gained insight, may help you in developing better problem-solving ability for your pupils.

If we wish our students to pass stereotyped examinations in which certain type problems occur, and this we all wish, then by all means let us teach a type problem, drill on the recognition and solution of the type, and continue this up to the day of the examination. The students will probably do well on the examination and still feel rightly that mathematics is a useless subject. On the other hand, if we are interested in developing *problem-solving ability*, then the following must be stressed.

Insightful problem-solving is characterized by sudden discovery and high retentiveness. The solution is from "above," seeing the situation as a whole, recalling past similar situations, restructuring or reorganizing all the elements of the situation into new patterns, making analyses of the situation relating the elements to the whole situation, reformulating the problem, making generalizations, and abstracting common properties. To develop this type of learning, we must build many concepts through the use of concrete experience and encourage pupils to avoid wild guesses and to seek relationships, to seek to complete configurations, to select functional and pertinent elements related to the entire situation, to frame hypotheses, to ask pertinent questions, ever to generalize their partial solutions, and to check every answer.

[6] T. A. Bickerstaff, "Problem No. E1086," *The American Mathematical Monthly*, LX, No. 9 (November 1953), 626.

# *Improving the learning of mathematics*

KENNETH P. KIDD, *University of Florida, Gainesville, Florida,*
*points out that learning is a complicated process and goes on to analyze this process for the classroom teacher.*

PROBABLY EVERY mathematics teacher has asked himself the question, "How can I bring about improvements in the teaching of mathematics?" An answer to this question presupposes a clear conception of the desirable outcomes of mathematics instruction. Let us assume that the test of competence in mathematics is the effective use of the skills and concepts of mathematics in problem situations which involve quantitative and spatial relationships.

Many teachers are convinced that attitudes, methods of thought, and habits of work which are developed in the classroom should be of as much concern to teachers as the selection and organization of the mathematics content to be studied. These teachers believe that the approach to the problem of improvement of instruction should be through a careful consideration of the learning process and of how to direct this process.

The classroom teacher hearing conflicting ideas from psychologists regarding the nature of the learning process has had difficulty in forming a clear conception of the factors affecting learning. He runs into further difficulty as he observes that learning is a complicated process and that there are probably many factors at any one time affecting learning. It will be the burden of this discussion to assist the teacher, to some degree, in overcoming these difficulties. Three major principles which seem to be common to the various theories of learning will be stated, and ideas relative to the implementation of these principles will be presented.

## I. INSTRUCTION SHOULD BE ORGANIZED AROUND THE DEVELOPMENT OF THE DESIRE TO LEARN

Over the past three decades there has been a significant shift in the thinking among psychologists relative to the importance of motivation and goal-seeking in the learning process. Kilpatrick expresses the stand of many psychologists in the following statement: "Learning goes on best in the degree that the individual himself sees and feels the significance, to his own felt needs, of what he does."[1]

The mathematics teacher must carefully maintain a balance between a logical sequence of ideas to be learned and a sequence based upon the concern of the students. Every teacher has realized the futility of trying to teach a logically arranged body of relationships to students who have no concern or interest in what is under consideration. Furthermore, probably every teacher can long remember instances of a remarkable lift in interest and enthusiasm which suddenly appeared in an individual student or even in an entire class when something happened to make the learning purposeful. What can be done to cause students to want to learn?

*Breathe life into the abstractions of mathematics.* Abstract ideas appear meaningless and lifeless to one unless he can see the relationship between these ideas and his own background of knowledge and experience. Much can be done to bring mathematical

[1] W. H. Kilpatrick, "The Philosophy of the New Education," *School and Society*, LIV (November 29, 1941), 482.

ideas to life for the students. Many teachers plan the development of mathematical concepts around simple and appropriate illustrations, and use very real applications of the concepts once they have been generalized. These teachers have a knowledge of the use of mathematical ideas in the school and community and have readily available a variety of clippings, models, reference books, and instruments. In fact, their classrooms with interesting displays of selected materials seem to tell the students that the mathematical ideas being studied "make sense."

A few illustrations are: (1) Fractional divisions on ruler and circular discs cut into equal parts to develop the meaning of common fractions, (2) canvass of various business activities in the community to secure data relative to measurement, (3) graphic representation of statistical data from community, county, state, nation, and world to put across the point that graphic methods may be used more effectively than many other methods to portray a quantitative idea, (4) stop watch, pebble, and steel tape to illustrate that the phenomenon of the falling pebble is described by $S=\frac{1}{2}gt^2$, and (5) Fahrenheit and Centigrade thermometers read in locations having varying temperatures to *verify* the formula $C=\frac{5}{9}(F-32°)$.

*Allow each student to gain some measure of success, and recognize his achievement of it.* Successful accomplishment of a task that has required genuine effort acts as a tonic for further effort. On the other hand, continual failure tends to deaden the desire for further effort, as the student reaches the conclusion that his achievement of success is hopeless. For example, arithmetic games that, over and over again, label certain students as failures, and tasks or assignments requiring more mature levels of response than these students are able to give, have contributed much to poor achievement in arithmetic. The following are suggestions for carrying out the foregoing directive:

1. Learn as much as possible about the varied capabilities of students

2. Differentiate learning activities for students on the basis of their capabilities and interests. Some measure of differentiation may be achieved through the use of projects, more difficult applications of ideas being studied, optional topics which are not basic to further work in the course, a browsing corner in the room, mathematical puzzles and paradoxes, and student reports

3. Exercise patience in order to encourage those who have given up hope.

*Provide challenging problem situations.* Many teachers have experienced upsurges in class interest and effort when students have become curious, puzzled, or doubtful about a problem situation. Typical questions come forth as interest heightens: How did he do that? Is that *really* true? How does it work? Which answer is correct? I don't see why it should turn out that way. In order that a situation present a challenging problem to students, one or more of the following elements must be usually present: newness, apparent inconsistency, and uncertainty as to exact answer.

The situation should pose a problem difficult enough to cause the student to put forth genuine effort, but generally not beyond his ability to realize success. Often situations cease to become challenging because the teacher gives too much assistance, leaving little opportunity for the students to experience the thrill of discovery. Students are not all challenged to the same extent by a particular situation and must be treated *differently* as they meet the challenge.

*Help students to feel a personal interest in the class.* One tends to look approvingly upon, or to be proud of, his own possessions, ideas, or actions. One is also more prone to accept responsibility for carrying out his own decisions than for carrying out decisions of others. Teachers should consider having students share in class decisions such as those relating to (1) statement of objectives, (2) selection of proj-

ects or optional activities, (3) formulation of acceptable criteria such as those for judging the quality of graphs, and (4) planning activities.

Students gain in many ways from working together in groups on the preparation of a mathematics exhibit, assembly program, classroom dramatization, panel discussion, or debate. The gains are augmented if the student group can be made to feel some measure of responsibility for the success of its undertaking. This feeling, or attitude, is generally more pronounced whenever the teacher steps into the background and lets the students make decisions and be in charge of presentations. Even the least capable student may usually be encouraged to accept responsibility if through working on some project which is different from the others he is in the position of having the rest of the class look to him for his superior understanding of an idea. And teachers sometimes wonder why many of the least capable mathematics students do not appear to get a thrill out of explaining an arithmetic problem or the proof of a geometry proposition which was assigned to *all* on the previous day!

Individuals generally seek to be useful or to be needed by others. Teachers can help students gain this feeling of being needed by using their ideas or by using student-made equipment in the classroom.

*Help students evaluate and accept responsibility for own progress.* The greatest growth in the development of independent work habits comes when the students evaluate and accept responsibility for their own progress. By means of self-administered diagnostic tests, teacher-made tests, and conferences with his teacher, a student should have a knowledge of his own strengths and weaknesses. He should have in mind fairly definite goals toward which he will work and see that progress toward these goals is both possible and desirable. Students must have time to work toward their goals and the teacher must be ready to help them to evaluate their progress.

### II. THE LEARNER MUST BE ACTIVELY ENGAGED IN THE LEARNING PROCESS

*Learning is an individual matter.* Learnings take place as the learner encounters a situation and reacts to this situation. The learnings are functions of both the characteristics of the situation and the nature of the learner. The teacher is the one charged with the responsibility of facilitating desirable learnings. He must find out about the nature of the learner and set the stage for learning by arranging to have him react to certain situations. The important idea is that the *learner himself must react*—he is the one who must be *actively engaged* in the learning process. There is no substitute in the classroom for the student's *thinking for himself!*

Abstractness and generality are characteristic of mathematics. It is important that mathematics teachers realize that students are at various levels in their ability to deal with these abstractions. Some students can handle quantitative ideas only when related to the more specific and concrete, while others can think through problems using the most concise, adult-developed symbolism and procedures. For example, students, when *first* confronted with a problem involving a two-figure multiplier like finding the cost of 12 loaves of bread at 17¢ each, differ in the way in which they may attack the problem. Some may be unable to arrive at the correct answer to the question at all; some may be able to arrive at the correct answer by working with 12 piles of coins; some may add twelve 17¢'s; and some may notice that 10 loaves would cost 170¢ and 2 more loaves would cost 34¢ more, making 204¢ altogether. In the beginning algebra class, students also display wide variations in levels of abstract thinking as can readily be seen if they are asked to give the value (in cents) of 7 nickels and then of $x$ dimes and $y$ nickels combined.

It is important that teachers not expect students having inadequate experience with specific number ideas to operate successfully and with understanding when

thrown abruptly into the manipulation of the most abstract symbolism. They must take care to provide situations whereby the learners can individually progress toward the more mature ways of quantitative thinking. This means that the learning activities must be appropriate to interests, needs, and understanding levels of the students. Students who have had limited experience with some new mathematical idea (such as fractions) will have much greater need of specific concrete illustrative materials; for students who have had successful experience with simple illustrative materials, it may be desirable to place emphasis on the development and use of the abstract symbolism.

*The climate of the classroom must cause each student to want to participate.* Reaching conclusions, even by adults, is not always a systematic process which avoids errors. If students think for themselves errors will be made and corrected. It is of very great importance, therefore, that the teacher make every effort to cause the students not to be afraid to make errors. To do this, mutual respect and trust must exist between students and between students and teacher. Holding respect for a person means that one will consider that person's human worth and not neglect him; that one will entertain that person's ideas and not be hasty to reject ideas which do not coincide with his own; and that one will be patient with sincere efforts and not ridicule incorrect responses or criticize errors.

Many teachers have taught the same mathematics courses so many times that they no longer learn as they teach. They have largely memorized the answers and methods of solution. It is easy for these teachers to become impatient when students are not quick to respond. Many wise teachers, however, encourage students to delve into topics which are new to the teachers. These teachers, by honestly assuming a position of co-learner with their students, do much to get wider participation and gain respect of the students. The following are suggestions: solving mathematical puzzles; learning how the Romans multiplied; learning to use an abacus; passing the Boy Scout test for surveying merit badge with one's students; learning about electronic computers, 3-D movies, or duodecimal number system; and finding out what units and instruments of measurement are used in the community. A great deal of curiosity may be aroused in the pursuit of such an idea if the final results of investigations are unknown even to the teacher. For example, in World War II some teachers taught weather-forecasting by using a series of surface maps which were two or three years old. Students analyzed a map, for example, for January 4, 1941, and from this prepared a forecast for January 5. Then the data for January 5 were brought forth, forecasts were checked, and students were told why they should have forecasted clear skies for St. Louis with NW winds rather than drizzle with SE winds. When some instructors adopted a practice of using current maps and made forecasts along with the students, the interest of the students received a lift. The students could see their instructors acting in situations similar to their own, since the instructors had no copies of the next day's weather in their notebooks!

*The reactions or thought processes of the learners warrant careful consideration.* It is not enough to know that the student has obtained the correct or the incorrect answer to a problem. A knowledge of the ways by which students in a classroom arrive at an answer may be of value in many respects. When a student is not able to answer correctly, an examination of the thought process often clearly points to what is causing the difficulty. Does the student know the meaning of $6 \div \frac{1}{2}$? Can he show with sticks and cardboard the meaning of the particular theorem being studied in solid geometry? Has the student memorized statements that are meaningless to him? An examination of the students' thought processes should reveal the levels of understanding and maturity of thinking in the classroom and furnish the

teacher with a basis for selection of activities which will help students toward fuller understanding.

It may be just as important to know that a certain procedure is correct as it is to carry out a procedure correctly but not be positive of its correctness. Certainly, to judge that a response has been made correctly one must have a *definite reason* for the response. Consideration of this reason may be prompted by the question, "Why?" On the other hand, the teacher who assumes the role of judge and hands down verdicts of "right" or "wrong" as responses are placed before his throne is encouraging the guessing of answers and discouraging the development of habits of independent work.

Some teachers encourage their students to work on their own a great deal in an effort to have them make discoveries of mathematical relationships and processes. These teachers find it profitable to bring to the attention of the class the various relationships or processes arrived at by the several students. By doing this, students may compare and select the most useful ideas. However, all may not always select the same method of thought. When several ways of attacking a problem may be equally effective, it is considered unwise to require stereotyped patterns of thought.

### III. THE LEARNER MUST GIVE MEANING TO WHAT IS BEING LEARNED

Much is said today about teaching mathematics in a meaningful manner. There is also accumulating a body of experimental evidence which supports the hypothesis that when the learner is able to give meaning to what he is attempting to learn he is able to learn just as easily, to retain more of what is being learned, and to make greater use of what is being learned.

Thiele[2] found the learning of the addition, subtraction, multiplication, and division facts proceeded more rapidly if the students were led to generalize number relations than if the facts were taught by drill only. Upon testing students one year after the finish of an elementary algebra class, Layton[3] found that these students could remember only one-third of the material known at the end of the course. Howard[4] found that students who were drill-trained in arithmetic also had high rate of loss of skills. He also found that emphasis on both meaning and practice resulted in very little loss.

How can we teachers know that mathematics is being taught meaningfully? New ideas take on meaning for the learner as he *relates* these ideas to his past experience. The words, "kuz," "sogl," "gib," and "pej" or the symbols $\flat\flat$, $\natural\flat$, seem absolutely meaningless to us because we can find nothing in our experience to which to relate them. Would it not be reasonable to assume that words such as "one," "two," "minus," "plus," and "coefficient," and abstract mathematical symbols such as $+$, $\div$, $\frac{3}{4}$, $x^2-3=0$, log 32, and $\int x^2 dx$ are also meaningless to one until he can *relate* them to ideas which he has previously experienced?

Having had background experience, however, is only part of the story. The important idea is that the new experience must be *related* to the old. The algebra student, for example, has had much experience with the symbols and operations of arithmetic. At the same time, he usually encounters much difficulty in beginning algebra because of his inability to *relate* the symbolism and operations of algebra to those of arithmetic. It will be the burden of the next few paragraphs to suggest ways of helping the student to relate mathematical ideas.

[2] C. L. Thiele, "The Mathematical Viewpoint Applied to the Teaching of Elementary School Arithmetic," *Teaching of Arithmetic, Tenth Yearbook*, The National Council of Teachers of Mathematics, pp. 215-24.

[3] E. T. Layton, "The Persistence of Learning in Elementary Algebra," *Journal of Educational Psychology*, XXIII (January 1932), 52.

[4] Charles F. Howard, "Three Methods of Teaching Arithmetic," *California Journal of Educational Research*, I (January 1950), 28.

*Experience with specific, concrete mathematical ideas precedes understanding of definitions and symbolism.* Symbols of mathematics, much like the symbols of our written language, are used to express ideas. Rules for the manipulation of these mathematical symbols are analogous to the syntax or grammar of our language. Before placing emphasis upon learning the addition facts, the students should have had experience in counting—using number symbols to record "how many"—and in thinking through situations of the types, "so many and so many more," "how many altogether?" Vivid illustrations, dramatization of real situations, consideration of real quantitative problems, and a variety of printed reference materials, charts, models and instruments are all important to the teacher who would have the students experience mathematical ideas.

Many students are hastened too rapidly from the concrete, specific, or definitional basis of working with mathematical expressions to the procedures used for most concise operations. Consider, for example, the inability of students to understand the idea expressed by the symbol "$\div$" in $8 \div \frac{1}{2}$, or the parenthesis in $3a-(a-2)$, or the exponent 3 in $(2\ ax^2)^3$. Exercises such as the following might be provided to show how the coefficient and exponent may be used to gain conciseness of expression.

Express the following in more concise form:

$a \cdot a \cdot a + b \cdot b =$

$b \cdot b + b \cdot b + b \cdot b =$

$(3x)(3x)(3x) =$

$(2a+b) \cdot (2a+b) \cdot 3a =$

$3(a-b)+3(a-b)+3(a-b)+\ldots+3(a-b)$ where $3(a-b)$ appears $n$ times.

Why not also have students do examples of multiplication of monomials by using definitions and basic principles before jumping into the rules? For example,

$$\begin{aligned}(3a^2b)(2a^3b^2) &= (3 \cdot a \cdot a \cdot b)(2 \cdot a \cdot a \cdot a \cdot b \cdot b) \\ &= 3 \cdot 2 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \\ &= 6a^5b^3.\end{aligned}$$

*Attention given to the main ideas of mathematics will help students to relate what would otherwise be isolated facts into a coherent structure.* Mathematical ideas fit together to form a unified structure. Many ideas, however, are considered one by one in a fragmentary manner, and these ideas do not fall neatly into place without continuous effort at creating this coherent structure. For this reason the statement has often been made that students of mathematics lose sight of the forest because of the trees. It is important that the students take airplane rides over the mathematical forest and examine the main features of the terrain—these features being the unifying concepts and basic principles.

In dealing with the integers of arithmetic, for example, some of these main features which may be used to relate what would otherwise be fragmentary, unrelated processes are: (1) counting as a process of putting into one-to-one correspondence a group of like objects with a sequence of number names or symbols whereby the last name or symbol used expresses the "how many" property of the group of objects; (2) the scheme of grouping objects into groups of ten with the group sizes—whether tenths, ones, tens, hundreds, and so forth—for a particular number symbol being determined from the position which that symbol holds in the number; (3) the function served by zero as an "empty" symbol; (4) the ideas of regrouping of groups of like objects implied by the four operations; (5) the existence of the associative and commutative principles in the regrouping process of addition and multiplication; and (6) the development of the various algorisms as mature concise methods of regrouping.

Often a teacher or textbook suggests that students follow a certain procedure or form in order to achieve uniformity or to avoid other procedures which easily lead

to confusion and errors. Such procedures include: (1) The various algorisms of arithmetic, (2) the positive and negative directions of the axes in the Cartesian co-ordinate system, (3) the type of decimal mark to use in writing place value fractions (usually called "decimal fractions"), and (4) the keeping of only one pair of = marks in any one horizontal line when solving algebraic equations. Emphasis upon basic concepts and principles would necessitate the students' distinguishing between principles and such purely arbitrary procedures. It would also necessitate the avoidance of such words as "cancel," "transpose," "cross-multiply," and "eliminate," which tend to disassociate procedures from authorization for the procedures.

By considering related ideas together, it is often possible to bring several ideas under one broad generalization and show that each of the related ideas is a special case of the broad generalization. The rule of likeness is an example of a general idea that has specific applications in each of the following instances:

$34+5=$ $3a+2b+a-b=$

$\frac{1}{2}+\frac{1}{3}=$ $\$3.75-\$.86=$

Add 1 ft. 3 in. to 1 ft. 7 in.

In each of the six figures at the right there is a definite relationship between the measure of angle $x$ and the number of degrees in arcs intercepted on the circle by the sides of the angle. One general relationship may be written which is broad enough to include all six instances.

Many of the special products in algebra can be shown to be special cases of the more general product of

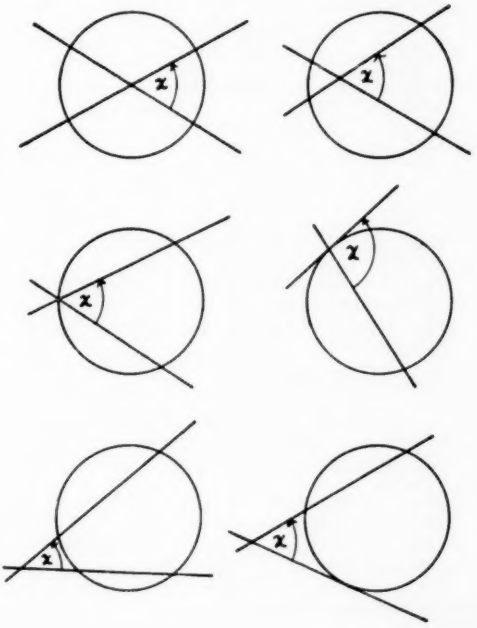
$$(a+b)(c+d)=ac+ad+bc+bd.$$

Some algebra teachers start with the development of this general form and then as this form is applied the students discover the conditions under which the 4-term product may be condensed into one of 3 terms or of 2 terms.

When a relationship has been established, it is a good policy to spend some time examining a few special cases (including limiting positions) under which the proposition holds. In this way one may discover that some relationship previously learned is in reality a special case of the more general relationship. For example, the trigonometry class may find that the Pythagorean theorem is a special case of the more general Law of Cosines.

Mathematical processes might also be related by using and comparing different approaches to the solution of a problem. For example, in finding distance across a stream, one might make a scale drawing, use trigonometric functions, or set up two similar triangles on the spot and make use of a proportion. Under what conditions might one method be selected over another? In what way do the methods employ identical mathematical ideas?

*Mathematical concepts and processes also take on meaning for the learner to the extent that he is able to use them to solve problems*



*or to interpret environment significant to him.* Giving meaning is a process of relating ideas. New mathematical ideas being learned must be related to other mathematical ideas previously learned. Yet still another test of how meaningfully mathematics has been taught is the success that students have in *relating* mathematical ideas to specific and significant phases of their experience.

It is a very sobering experience for a mathematics teacher to talk with teachers who teach his students in shop, home economics, chemistry, or physics. Do his students relate class work with fractions to the use of ruler in marking off $2\frac{3}{4}''$ on a board? Do they recognize that finding $\frac{1}{2}$ of a recipe which calls for $\frac{3}{4}$ cup of an ingredient involves finding $\frac{1}{2}\times\frac{3}{4}$? Do they apply their skill in solving equations effectively to the proportions of chemistry or the formulas of physics which do not appear in the same form as those on which they have practiced?

Many mathematics teachers supplement textbook exercises by considering problem situations which are significant to their students. In such situations the "problem" may often be stated in somewhat unfamiliar terminology, may contain extraneous factors, or may be stated much less concisely and less clearly than textbook problems. The students may also be required to select and obtain relevant data, decide how to attack the problem rather than follow a stereotyped example, and test their answers rather than look in answer books.

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## Parabola

*By Hooper Reynolds Goodwin, Bethel, Vermont.*

This curve I'm plotting? A parabola.
This point is called the focus; it's the point—
Oh, no, not an ellipse. Ellipses have two foci:
Here, I'll show you one I've drawn.
You see the difference. These two lines of the parabola,
They stretch out wide and wider,
"World without end," as preachers say.
(I don't know what they mean; perhaps they don't);
But you see how it goes.

There was a man—Sir Isaac Newton, I believe it was—
Who had the notion a parabola was an ellipse,
Its other focus at infinity.
You may not understand just what he meant;
You have to sort of take the thing on faith.
The keenest scholar can't quite picture it, you know.

I've often thought,
It might be called a symbol of man's life;
A curve of ever-widening sweep.
And here in this world
Is the focus we may call, say, temporal interests,
Food and drink and clothes . . .
But yet it cannot be that this is all;
For out beyond the reach of sight must be
Another point, a heavenly focus, see?
'Round which the sweeping curves of human life
Complete the ellipse,
Fantastic? Well, perhaps,
But yet the more I think of it . . .

And here—
Another thing I've often thought about:
Suppose we draw here two parabolas
With axes parallel, and let the arms cross—
"Intersect" the word is—at this point.
Now if there be a focus
Somewhere out beyond the bounds of space,
And these are two ellipses,
As Sir Isaac thought they were,
Why, don't you see, they'll intersect again
Somewhere out there.
Just as two lives that once have crossed,
Then gone their separate ways,
And one has disappeared long since into the void of death
May—but who knows? It's just a thought . . .

Well, come again; I don't get callers often.
They don't see much in old folks nowadays,
And when a man's not only old, but got his head
Stuck always in a book of "Analyt!"
Young people think I'm queer; they can't see why
A man that doesn't have to study graphs
Should plague his head; don't understand that such
Dry, dull things as a parabolic curve
May bring up mem'ries of a face that's gone.—

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# *The function concept in secondary school mathematics*

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*A needed discussion of an important concept in mathematics. The editor would welcome other discussions of important ideas to be studied in secondary mathematics.*

THE ADOPTION of the double-track plan for high-school mathematics should provide vital, meaningful, and needed mathematical experiences for those whose ability or interests render the sequential courses unsuited. But the plan should do more; it should provide an opportunity to repair some of the watering down and emasculation which have taken place in the sequential courses in our efforts to fit them to the wrong pupils. If we fail to seize the opportunity, provided by this curriculum trend, to strengthen and revitalize the offerings aimed at the college preparatory student, we shall have lost a great part of the merit of the idea.

This does not imply a return to the curriculum of thirty years ago. Quite the contrary, the sequential curriculum, granting that its function is a preparatory one, is probably in just as great need of a thorough rethinking and possibly complete overhauling as is the offering for the nonpreparatory student.

There are doubtless many improvements which can and should be made within the present structural context of the material. It is the purpose of this paper to propose one such change, namely the earlier introduction of the concept $y=f(x)$.

One of the most powerful and useful concepts in elementary mathematics is that of functionality and dependence. Yet, there is ample evidence that it is not being exploited sufficiently. All too frequently the typical college freshman, when asked to graph the quadratic equation

$$ax^2+bx+c=0$$

will respond with a parabola cutting the $x$-axis at the roots of the equation. This could be taken as evidence that the graphic solution of the quadratic equation had been very well mastered. However, there are more serious implications. The fact that the graph of the equation consists of two straight lines parallel to the $y$-axis—

$$x=\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } x=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

is met with disbelief by many students. The graphic solution of the quadratic equation has been well learned as a rule of thumb procedure, but a lack of understanding of the significance of what has been done is evident.

There seems to be no appreciation of the fact that the graphic solution actually utilizes the notion of graphing two functions of $x$: $f(x)=ax^2+bx+c$ and $F(x)=0$; then finding those values of $x$ for which both functions have the same value. One wonders whether any points on the parabola other than the intersections with the $x$-axis have significance for the student.

Under such circumstances the main justification for studying the method is missed. The writer was impressed recently with the same shortcoming in a differential calculus class. In the study of maxima and minima, the class had a problem which required the determination of maximum and minimum values of the function $-ax^3+bx^2+cx+d$. Every member of the class differentiated correctly and set the derivative equal to zero. At this point, following a procedure which had been learned in algebra, with or without understanding its true significance, the signs were changed in the resulting equation in order to eliminate the leading negative coefficient. The solution of this equation did give the correct critical points. At this point, the trouble began. The class had not yet studied the test for maximum and minimum values which involves the second derivative, so values above and below the critical points were used to determine the change of signs. These values were used in the derived equation where all signs had been changed, rather than in the original. The class obviously had not learned to distinguish between the function, $f(x)$, and the equation, $f(x)=0$. To say the least, the misapprehension seemed to be present that, since $f(x)=0$ and $-f(x)=0$ have the same roots, $f(x)$ and $-f(x)$ are identical.

From the pedagogical point of view, the two most obvious jumps from arithmetic to algebra are the extension of the number system beyond positive rational numbers and the further generalization of the number concept to include literal numbers. The formal introduction of the idea of $y=f(x)$ could well be a part of the development of the concept of the use of letters to represent numbers. The pupil is taught that a letter is used to represent a number. These letters are of two kinds; the letter may represent a number whose value does not change; it may have a known value, such as $\pi$, or it may be unspecified but unchanging throughout a given discussion. Or the letter may represent a number whose value does change. The usual approach to equations can serve only to confuse rather than clarify the distinction between constant and variable. We talk about unknowns in an equation. We have solved the equation when the unknown becomes known. This sounds very much like a constant yet the pupil is expected to think of it as a variable, in spite of the fact that in an equation such as

$$2x+3=x-4$$

we cannot properly think of $x$ as a variable unless we consider the equation as an equality of two functions. The *variable* $x$ may take on any value in each of the two functions, and the solution consists of finding that value which makes the functions equal.

In an effort to get a fair picture of current practices relative to the development of the concept of a mathematical function, a dozen algebra texts of recent date, six first-course and six second-course, were examined. Only three of the twelve employed the notation $y=f(x)$. Two of these introduced the idea casually in connection with graphing. Eleven of the twelve nowhere advanced the idea that an equation is an equality of functions. Four of them contained no treatment of conditional versus identical equations. None of the twelve utilized the idea that in an identity both members represent the same function, while in the conditional equation each member represents a different function.

Recent and current literature is replete with exhortations to strive for meaning, not that manipulative proficiency carries a stigma, but manipulative proficiency devoid of meaning and understanding is more or less time wasted. It is in the interest of teaching for meanings and deeper understanding that this paper proposes an earlier introduction and greater utilization of the function concept.

The idea of a function should be developed before the introduction of equations. This can well be done in connection with the idea of the general number. We use the letter $x$ to represent a quantity which is

capable of taking on any values we desire. Just as $x$ represents a variety of values, so $2x$, $3x-5$, etc. also take on a variety of values, but their values depend on and are determined by the behavior of $x$. This will enable us to introduce the equation as an equality of functions. The usual procedure consists of developing the idea that an equation consists of two quantities which are equal but unknown, since the $x$ is unknown. The proposed procedure would develop the idea that an equation (conditional) consists of two functions, each capable of changing, their values dependent upon the behavior of the variable $x$. Under the first concept of an equation, its solution consists of finding the unknown $x$ such that the two members of the equation will be equal. Under the function approach to the equation, no assertion of equality is made. A question is posed: Under what condition will these two different, varying quantities take on the same value?

Much of the merit of the proposed approach depends upon the early introduction of rectangular co-ordinates. This, within itself, is no drawback. Such work should be an outgrowth of work with directed numbers. An integral part of the development of the function concept should be the graphing, not of equations, but of functions.

Under which approach to the equation will the steps in its solution be the more easily rationalized? In either event our objective is to convince the pupil of the appropriateness of the axioms we use. The axioms of addition, etc. are usually phrased in terms of preserving *an* equality. Yet the pupil all too often gets the erroneous notion that he is preserving *the* equality he started with. By graphing the pairs of functions in the sequence of equivalent equations obtained in deriving a solution, the pupil cannot fail to see that the successive pairs of functions are quite different, yet they do have something in common. The value of $x$ which makes the pair of functions equal is the same at all steps. Thus, we do not merely change the form of the equation. We are, rather, replacing the original relationship for more easily recognizable ones until ultimately the solution is obvious. When we reach the point where we have $x=c$, we still have two functions, but we now are confident in stating for what value of the variable $x$ the two functions are equal. Under the proposed approach, systems of equations in two unknowns offer nothing new. We are merely giving our two functions the name, $y$.

Dependent and inconsistent equations hold no mysteries if the pupil thinks in terms of functions. Our answer to the question, "For what value of $x$ are these functions equal?" is "Any value" in the first case and "No value" in the second. The graphing of the functions makes such a conclusion meaningful. The misunderstanding referred to earlier surrounding the graphic solution of the quadratic equation would not exist if the pupil were fully aware of the fact that we are doing exactly the same thing as is done when two simultaneous linear equations in two unknowns are solved graphically. We are looking for the intersection of the graphs of two functions—the quadratic function and the function $f(x)=0$.

The distinction between a conditional equation and an identical equation, when approached from the standpoint of functions, is quite easy. In the former we investigate the conditions under which two functions will be equal, while in the latter we investigate the possibility that two functions are actually two forms of the same function. Thus the usual injunction not to cross the equality mark in proving an identity becomes a reasonable restriction.

Perhaps the greatest benefit to be derived from the function approach to equations is in connection with the solution of verbal problems. Let the following problem illustrate the point.

An automobile cooling system holds 5 gallons. If the cooling system is filled with a 20% alcohol mixture, how much of the

mixture must be withdrawn and pure alcohol added in order to obtain a 35% alcohol mixture?

The most obvious variable in this situation is the amount of the mixture withdrawn. Call this variable $x$. (The fact that the range of values is restricted to $0 \leqq x \leqq 5$ need offer no difficulty. In fact we should emphasize the idea that in most practical situations the range of our variable is restricted.) The final mixture must contain $.35(5)$ gallons of alcohol. The alcohol which remains after withdrawing $x$ gallons of the mixture is certainly a function of the amount withdrawn, namely $.20(5-x)$. The alcohol which must be added is also a function of $x$:

$$.35(5)-.20(5-x).$$

Thus, $x$; $.20(5-x)$; and $.35(5)-.20(5-x)$ are all variables, the value of the latter two depending on the value $x$ assumes. Since we are to replace the mixture withdrawn with the same amount of pure alcohol, the alcohol added is not only $f(x)=.35(5)-.20(5-x)$, it is another function of $x$, namely $x$ itself. These two functions of $x$ which represent the alcohol added are not, in general, the same. The problem requires that we find the condition under which these two different functions take on the same value. Thus the equation

$$.35(5)-.20(5-x)=x.$$

The analysis could be altered somewhat if we have developed the notion that a constant may properly be considered a function of the variable (a notion which certainly should receive attention). We may then consider $.35(5)$, or the alcohol in the final mixture, a function of $x$. The amount of alcohol remaining after the withdrawal, plus that added, is also a function of $x$ and is also the amount of alcohol in the final mixture. The problem requires that these two functions be equal, which leads to the equation

$$.35(5)=.20(5-x)+x.$$

It is believed that the approach to equations described herein can be a great aid in many problem situations.

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## Spirals

Spin a spiral 'round my head,
Embrace me once, proclaim me dead.
From spirals starting at my birth
In imitation of the earth
I spiraled till I now go, brave,
To seek the spirals of the grave.
Lay the conch shell at my feet,
And strew sunflowers on the street;
Lay me on my back that I
May watch the nebula-filled sky.
As final rite then lay me down
Wrapped in the mummy's spiraled gown;
Spin a spiral 'round my head,
Embrace me once, proclaim me dead.—

*Nancy, Main, 640 C Street N.E., Washington 2, D.C.*

## • DEVICES FOR THE MATHEMATICS CLASSROOM

*Edited by Emil J. Berger, Monroe High School, St. Paul, Minnesota.*

# *A model for explaining how latitude may be determined by making observations on Polaris*

*By Emil J. Berger*

One of the simplest ways of finding one's latitude in the Northern Hemisphere is to determine the "altitude" of Polaris. Tenth-grade students of plane geometry are usually quite fascinated by this application of one of the theorems which they study, and as often as not they are as interested in proving the method as they are in making an actual observation. In fact, from experience this contributor is able to say that they seem to derive more satisfaction from gaining an insight into the principles involved than in recording observations.

The theorem referred to above may be stated as follows: If two angles have their sides perpendicular right side to right side and left side to left side they are equal.

Illustrated in Figure 1 is a teaching aid which may be used to show how the fact contained in the statement of the theorem is applied and also to assist with the task of developing an idea which is equally important to the proof of the method but somewhat more subtle. The latter idea is that the direction of Polaris is assumed to be the same for all points of observation in the Northern Hemisphere. The experienced teacher knows that developing this concept with tenth-grade students is not easy. It is difficult because it involves the idea of an infinite quantity. The device illustrated in the figure will prove its worth in communicating this concept if one chooses to use it. Following is a list of the materials which are needed to produce it: one circular piece of plywood 12″ in diameter and $\frac{1}{4}$″ thick, one 180° protractor, two pick-up sticks, two pieces of round dowel each 9″ long and $\frac{1}{4}$″ in diameter, one $\frac{1}{8}$″ bolt $\frac{5}{8}$″ long, and a wing-nut to fit the bolt. The device is assembled in accordance with the following description.

$C$ is the center of the plywood circle. $EE'$ and $CN$ are respectively a diameter and radius which are perpendicular to each other. Both lines may be made with narrow strips of colored Scotch tape. $NP$ is a pick-up stick which is thrust into the edge of the plywood circle at $N$ in such a way that it is in the plane of the circle and appears as an elongation of line $CN$. The 180° protractor is glued to the board in the position indicated. $CD$ is a dowel which is fastened with a nail to the plywood circle at $C$ loosely enough so that it may be rotated as desired. Line $HH'$ is a pick-up stick which is attached to $CD$ at $T$ and is perpendicular to it at that point. It is in the plane of the plywood circle, and, since it is fastened to $CD$, moves with the dowel in such a way that it always appears as a tangent to the circle for all positions of $CD$. It may be fastened to the dowel by drilling a small hole through the latter and pulling the pick-up stick through. $P'W$ is another dowel. It is fastened to $CD$ at $T$ with a wing-nut in such a way that it may be adjusted in the plane of the circle. The entire device—all lines and moveable

parts—is supposed to be in one plane. In what follows this plane will be referred to as "the plane of observation."

When one seeks to find the latitude of one's location by observing the "altitude" of Polaris, one actually makes certain assumptions and depends on relations which are the result of these assumptions. If $T$ is a location from which an observation is made on Polaris, one of these assumptions is that the direction of $TP'$ is approximately the same as that of line $CNP$ regardless of the particular location of $T$. This assumption depends, of course, upon the fact that the magnitude of the distance to Polaris is very large. The device is designed to give concrete expression to this assumption and to help the student visualize the relations which result from this assumption. To develop this idea a bit further, suppose that after a particular point $T$ has been chosen, $P'W$ is adjusted so that it will be parallel to $CNP$. Then $P'W$ is perpendicular to $EE'$. Consideration of several different positions for $T$ will help clarify a student's comprehension of what is involved.

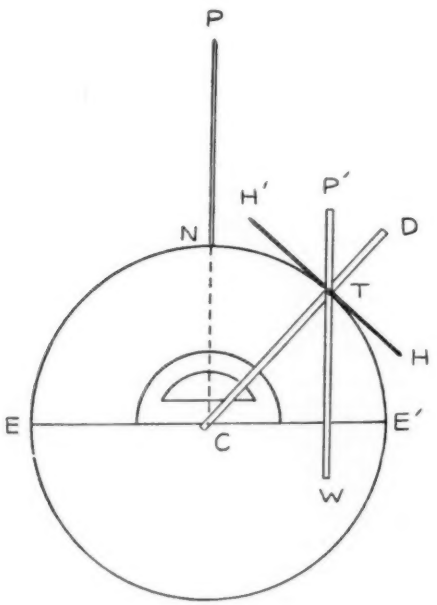


*Figure* 1

Another assumption that is made is that one's horizon (or horizon circle) lies in a plane which is tangent to the surface of the earth at the point of observation. In the model $HH'$ represents the horizontal line which lies in both the tangent plane and in the plane of observation. What is commonly called the "altitude" of Polaris is actually the magnitude of angle $H'TP'$ (also called the angle of elevation of Polaris). Since $EE'$ represents a diameter whose ends lie on the equator, and since $HH'$ is perpendicular to $CD$, and $P'W$ is approximately perpendicular to $CE'$, angle $H'TP'$ = angle $TCE'$. The last named angle is the latitude of $T$. Moving $CD$ around so as to vary the size of angle $H'TP'$ (always being sure to keep $P'W$ parallel to $CNP$) helps develop students' assurance that the method works and that it can be proved for all positions of $T$.

The device presented is based on the ideal assumption that the horizon (and therefore the line $HH'$) can be located. The only time that this is possible is when observations are made at sea, in which case the horizon may be thought of as the circle in which the water and sky seem to meet. While all this is true, the method and the proof suggested by the device are the same even if one is compelled to refer his measurements to an artificial horizon established with a bubble device. So even though geometry classes do not normally operate "at sea" the flavor of the application remains.

# *Semi-permanent chalk for teaching*

*Contributed by Robert C. Hoisington, Central High School, Minneapolis, Minnesota*

Probably every teacher who has ever taught graphing has at some time wished that he might have a squared cross-section blackboard which would be available when needed and which could be disposed of when not in use. To have the convenience of squared cross-section blackboard space, teachers often paint permanent graph charts on blackboards in their rooms. Occasionally some teacher solves the problem by scratching the desired cross-section markings on a blackboard with a sharp tool. Unfortunately both of these measures are permanent; the blackboards which are painted or scored are usually no longer useful for purposes other than graphing.

Some time ago this writer came across a clever little method for providing a class with a *semi-permanent* squared cross-section blackboard. Semi-permanent chalk will do the trick. Its preparation will be described presently. The special property which it possesses that makes it useful for the situation described in the foregoing is that marks made with it cannot be erased with an ordinary *dry* felt eraser while at the same time a *wet* cloth will remove all such marks cleanly.

To our knowledge semi-permanent chalk is not available commercially; however, it is not difficult to prepare. Anyone who wishes to make some can do so by observing the following steps:

*Step 1.* Prepare a stock solution. To a small amount of cold water (2 to 5 ounces) add enough sugar to produce a saturated solution. A saturated solution has been obtained when, after vigorous shaking and stirring, there is still a small amount of sugar left in the bottom of the container. Note that *cold* water must be used in the preparation of the stock solution. The reason for this is that the crystalline form of the sugar rather than the colloidal form is desired.

*Step 2.* Obtain a soft grade of chalk. Neither hard nor dustless chalk is satisfactory.

*Step 3.* Soak a number of sticks of chalk (colored chalk if desired) in a solution consisting of one part of the stock solution and three parts water. When the chalk ceases giving off bubbles, remove it from the solution and allow the excess liquid to drain away. In a short time the chalk will be ready for use.

Sticks of chalk which have been soaked in the fixing solution may be kept in usable condition for an indefinite period of time by storing them in an air-tight container such as a fruit jar. The fixing solution can also be preserved—at least for several months—if it is kept in an air-tight container.

If a squared cross-section graph chart is drawn on a blackboard with treated chalk, the chart can be used over and over for many different exercises, because the dry eraser which is ordinarily used to erase each separate graph will not disturb the semi-permanent chart. As already stated, when the chart is no longer needed it can be removed with a wet cloth.

The usefulness of semi-permanent chalk is not confined to drawing graph charts. Suppose, for example, that a class is studying a unit on circles. If a basic circle is drawn on the blackboard with semi-permanent chalk, it can be used again and again with many different exercises. The secants, chords, etc. that are needed to complete the figures for specific exercises may be drawn with ordinary chalk as needed and erased as desired.

## • HISTORICALLY SPEAKING,—

*Edited by Phillip S. Jones, University of Michigan, Ann Arbor, Michigan.*

# *Tying mathematics to its history*

*Contributed by Raymond L. Krueger, Wittenberg College, Springfield, Ohio*

Mathematics is so often accused of being dry and uninteresting. To me this is a real challenge for the teacher. Most of the content material *can* be taught by *just* a lot of good hard drill and practice. But as such it's no wonder students "moan and wail."

There's so much to teach and so little time. I'm for teaching less and doing it more thoroughly. This rapid skimming over a quantity of material may be all right for a lot of subjects, but it doesn't work in highly sequential mathematics; too many vital things are thereby only partially learned. The continuing student later pays the price. It is so often then well-nigh impossible to go back and master the preliminaries needed. No wonder originally eager mathematics majors soon become discouraged. They are confronted with too many gaps to fill.

So let us by all means teach, what we do teach, well. Anything which helps toward this end is all to the good. I'm reminded of several episodes out of the history of mathematics which tie in excellently with this: There are, of course, many others which the inquiring teacher will easily find, and use. Some are so classic as to be essential in the cultural background education of students, even if little is contributed thereby to their mathematical ability. I mean such items as (1) the bath episode of the discovery of Archimedes' principle, (2) the three famous problems of Greek geometry, (3) Zeno's paradoxes.

A fourth is the old story of the race between Achilles and the tortoise. This, used when we come to infinite geometric progressions with ratio less than one, really starts the class thinking (reference 4).

Another is Fermat's last theorem. This can be introduced in high school very nicely in connection with the Pythagorean theorem. I use it for our course in analytic geometry when we study the family of curves $x^n+y^n=a^n$ (see reference 5).

A sixth is the attempt of the Pythagoreans to suppress the troublesome discovery of the new type of number $\sqrt{2}$. It is a fine introduction to irrationals (see reference 6).

None of these have to be developed extensively, nor do they take any appreciable class time. But they do supply the occasional spice and zip that can be so stimulating. Some helpful references for these and other historical anecdotes are:

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6. Hooper, Alfred. *Makers of Mathematics*. New York: Random House, Inc., 1948. P. 74.

# *George Washington and mathematics education*

*Contributed by Edmund E. Ingalls, Albion College, Albion, Michigan*

The extract from Nicholas Pike's *New and Complete System of Arithmetic* on page 40 of the January, 1954, issue of THE MATHEMATICS TEACHER together with the comments on the advertising recommendations in *Palmer's Pocket Scale* on page 503 of the November, 1953, issue recalled to me an interesting mathematical letter written by George Washington to Nicholas Pike commending his book. I located this while doing research on Pike and his arithmetics in his home town, Newburyport, Massachusetts. It was printed on pages 327–32 of Mrs. E. Vale Smith's *History of Newburyport*, which was published in that town in 1854. Katherine M. Kuechle, librarian of the Newburyport Public Library, sent me this copy. She and her staff were most helpful at the time I sought this and other data about Pike and Michael Walsh, author of another early American arithmetic. The original is now in the "Papers of George Washington" collection of the Library of Congress.

The letter itself is self-explanatory. It contains comments which are quotable in discussions of some of our present mathematics-education problems:

Mount Vernon, June 20th, 1788.

Sir:—I request you will accept my best thanks for your polite letter of the 1st of January, (which did not get to my hand till yesterday) and also for the copy of your "System of Arithmetic," which you were pleased to present to me.

The handsome manner in which that work is printed, and the elegant manner in which it is bound, are pleasing proofs of the progress which the arts are making in this country. But I should do violence to my own feelings if I suppressed an acknowledgment of the belief that the work itself is calculated to be equally useful and honorable to the United States.

It is but right, however, to apprise you that diffident of my own decision, the favorable opinion I entertain of your performance is founded rather on the explicit and ample testimony of gentlemen confessedly possessed of great mathematical knowledge, than on the partial and incompetent attention I have been able to pay to it myself. But I must be permitted to remark that the subject in my estimation holds a higher rank in the literary scale than you are disposed to allow. The science of figures, to a certain degree, is not only indispensably requisite in every walk of civilized life, but the investigation of mathematical truths accustoms the mind to method and correctness in reasoning, and is an employment peculiarly worthy of rational beings. In a cloudy state of existence, where so many things appear precarious to the bewildered research, it is here that the rational faculties find a firm foundation to rest upon. From the high ground of mathematical and philosophical demonstration, we are insensibly led to far nobler speculations and sublime meditations.

I hope and trust that the work will ultimately prove not less profitable than reputable to yourself. It seems to have been conceded on all hands, that such a system was much wanted. Its merits being established by the approbation of competent judges, I flatter myself that the idea of its being an American production, and the first of the kind which has appeared, will induce every patriotic and liberal character to give it all the countenance and patronage in his power. In all events, you may rest assured that as no person takes more interest in the encouragement of American genius, so no one will be more highly gratified with the success of your ingenious, arduous and useful undertaking, than he who has the pleasure to subscribe himself, with esteem and regard,

Sir, your most obedient and very humble servant,

G. WASHINGTON.

Nicholas Pike, Esq.

It might, incidentally, be asked what it was there in the town of Newburyport which stimulated and encouraged these two men, Nicholas Pike and Michael Walsh, and made it the home of two early American arithmetics.

Newburyport was a thriving, prosperous seaport town with extensive and lucrative overseas and coastal trade. As an indication of the wealth of the town, the town gave to the support of the Revolutionary Cause at least a half million *pounds* in cash and goods. There was some manufacturing

as indicated by the fact that the first broadcloth made in the country was manufactured close by.

The demands for training for commercial pursuits, as well as for college preparation were met by the town "Grammar School." Since most of the Newburyport youth who were destined for the university attended Dummer Academy near Newburyport, a majority of the scholars in the town school were interested in "commercial" applications.

So it is not surprising to find that a large part of Pike's *Arithmetic* deals with simple and compound interest, annuities, insurance, exchange of money, and partnerships. In order that his book might contain the decimal system of coinage being proposed, the publication of his book was delayed until Congress authorized our present system of coinage.

Beyond this material there is some material on "plain" geometry and "plain" trigonometry as well as measurement of solids and an "Introduction" to algebra.

The book was popular partly for patriotic reasons. There was a natural reaction against British texts. Mr. John Scales, an editor of Manchester, New Hampshire, wrote later, "They (British texts) were faulty, heavy, obscure, and not at all adapted to conditions existing in America. The practical work of the schoolroom no less, perhaps, than his patriotism prompted him to make a change." The young republic had a "pound shortage" and was anxious to establish "home" industries. And there was need for a book which would give problems of local interest and application.

Nicholas Pike, too, merits a few comments. A Harvard graduate, he was appointed master of the third parish grammar school at a salary of 80 pounds in 1773. It is interesting to find that the school committee voted in 1780 "that the schoolmasters' salaries be raised in proportion to the rise of mechanics' wages, taking their salary in 1774 to be the standard."

He was also town clerk, a selectman, and a justice of the peace. Some of the people who came before him complained that he dealt with them as severely as he did with his scholars. He gave a sum of money to establish a scholarship at a school in Andover, Massachusetts, and, finally, Harvard honored him with a master of arts degree.

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## The Cooperative Committee on the Teaching of Science and Mathematics of the American Association for the Advancement of Science

At the annual spring meeting of the AAAS Cooperative Committee on the Teaching of Science and Mathematics held in Chicago, Illinois, John R. Mayor of the University of Wisconsin was elected chairman. Dr. Mayor, for several years the representative of the Mathematical Association of America on the committee, succeeds Morris Meister, principal of the Bronx High School of Science and representative of the National Science Teachers Association. Laurence L. Quill of Michigan State College, representing the Division of Chemical Education of the American Chemical Society, was elected to the newly created office of vice-chairman. Bernard B. Watson of the Operations Research Office of Johns Hopkins University, representing the American Association of Physics Teachers, was re-elected secretary of the committee. The National Council of Teachers of Mathematics is represented on the committee by George E. Hawkins of Lyons Township (Illinois) High School and Junior College.

## • MATHEMATICAL MISCELLANEA

*Edited by Paul C. Clifford, State Teachers College, Montclair, New Jersey, and Adrian Struyk, Clifton High School, Clifton, New Jersey.*

# *Company for Pythagoras*

*By Adrian Struyk*

Here are seven geometrical situations which are governed by the equation

$$a^n+b^n=c^n.$$

Seven different values of $n$ are involved: $2, -2, 1, -1, \frac{1}{2}, -\frac{1}{2}, \frac{2}{3}$. Perhaps some of our readers would be interested in matching these exponents with the situations to which they apply. The quantities $a$, $b$, $c$ are as described below.

1. The distances from a point on a circle to the vertices of an equilateral triangle inscribed in the circle.

2. The radii of three circles tangent to a given line and externally tangent to each other in pairs.

3. The lengths of the line-segments joining the vertices of an equilateral triangle to a point within the triangle if two of these line-segments form an angle of 150°.

4. The legs and the altitude to the hypotenuse of a right triangle.

5. The distances of two perpendicular lines from a given point in their plane, and the length of the shortest line-segment which contains the given point and is terminated by the two lines.

6. The dimensions of a rectangular block whose surface area is equal to the square of a body diagonal.

7. The bases of a trapezoid and the length of a line-segment drawn parallel to these bases from the point of intersection of the diagonals to one of the nonparallel sides.

# *Theme paper and ruler finale*

*By Adrian Struyk*

This note is the last one of four which present methods of sketching the conics, using a ruler and composition paper. The four notes do not exhaust the subject. There are various methods by which the conics may be obtained as envelopes. Then too, many of the theorems studied in plane geometry can be illustrated or applied, and relevant figures laid out, merely by drawing lines and counting spaces. To lengthen this series in an attempt to include more of such material would, however, very likely achieve little more than monotony. Hence we conclude by returning to the parabola, previously dealt with in a different way in the first of these notes.

In Figure 1 the points $P_0, P_1, P_2, \cdots, P_n$ lie on a parabola. They are obtained as follows: On the work sheet choose $n+1$ equally spaced rulings $l_0, l_1, l_2, \cdots, l_n$. $P_0$ is any point on $l_0$; $P_n$ is any point on $l_n$. Points $A_1, A_2, \cdots, A_{n-1}$ are marked on $l_n$ so that successive spaces $P_nA_1$, $A_1A_2$, $A_2A_3, \cdots, A_{n-2}A_{n-1}$ are proportional to the consecutive integers $1, 2, 3, \cdots, n-1$. Draw $P_0A_{n-1}$, cutting $l_1$ at $P_1$; draw $P_1A_{n-2}$, cutting $l_2$ at $P_2$; etc. Continue this process, finally obtaining $P_{n-1}$ on $l_{n-1}$ by

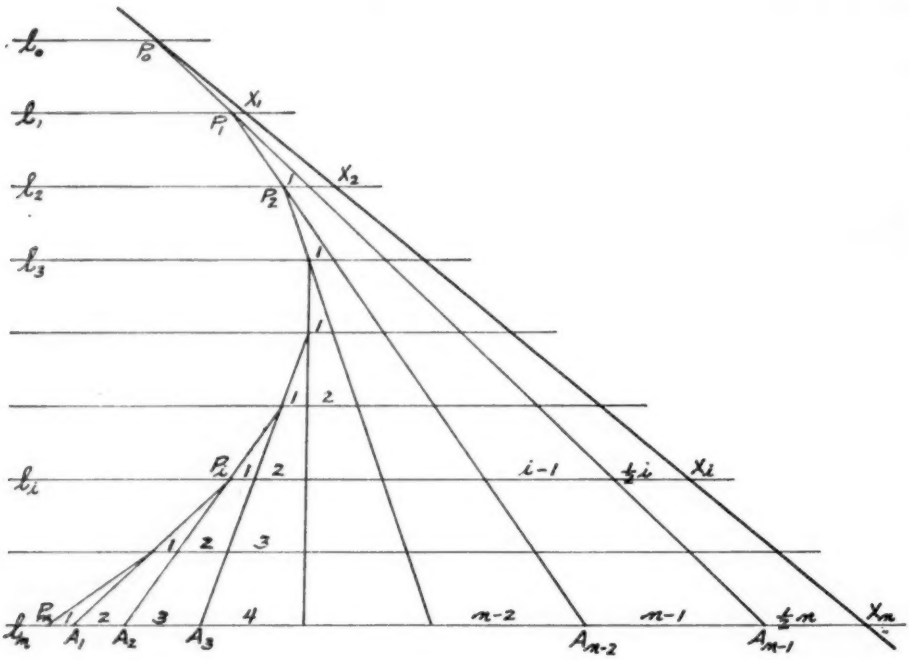


*Figure* 1

drawing $P_{n-2}A_1$. (Since the curve has not been drawn in Figure 1 the chord $P_{n-1}P_n$ is included for the sake of appearance.)

In order to demonstrate that the points $P$ lie on a parabola we make use of the line $P_0X_n$. Let us take $P_nA_1$ as the unit of length in expressing distances along the rulings. Then

$$P_nA_{n-1}=1+2+3+\cdots+(n-1)$$
$$=\tfrac{1}{2}(n-1)(n),$$

or

$$P_nA_{n-1}=\tfrac{1}{2}n^2-\tfrac{1}{2}n.$$

Now take

$$A_{n-1}X_n=\tfrac{1}{2}n.$$

Then

$$P_nX_n=\tfrac{1}{2}n^2. \qquad (1)$$

Each segment along $P_nX_n$ is the base of a triangle which is cut by a series of equally spaced parallels (the rulings). Hence Figure 1 as a whole consists of several sets of similar triangles. From a consideration of these it is plain that the pattern which was set up on $l_n$ and which produced the result (1) is repeated on any other ruling $l_i$, making

$$P_iX_i=\tfrac{1}{2}i^2.$$

Hence $P_i$ lies on the parabola through $P_n$, tangent to $P_0X_n$ at $P_0$, with axis parallel to the rulings.

The layout of Figure 2 is a corollary to that described above. In Figure 1 we observe that on each ruling $l_i$ the segment adjacent to $P_i$ is equal to the unit. Since the unit is of arbitrary length this simply means that these segments are equal to each other. Application of this fact yields results as illustrated in Figure 2.

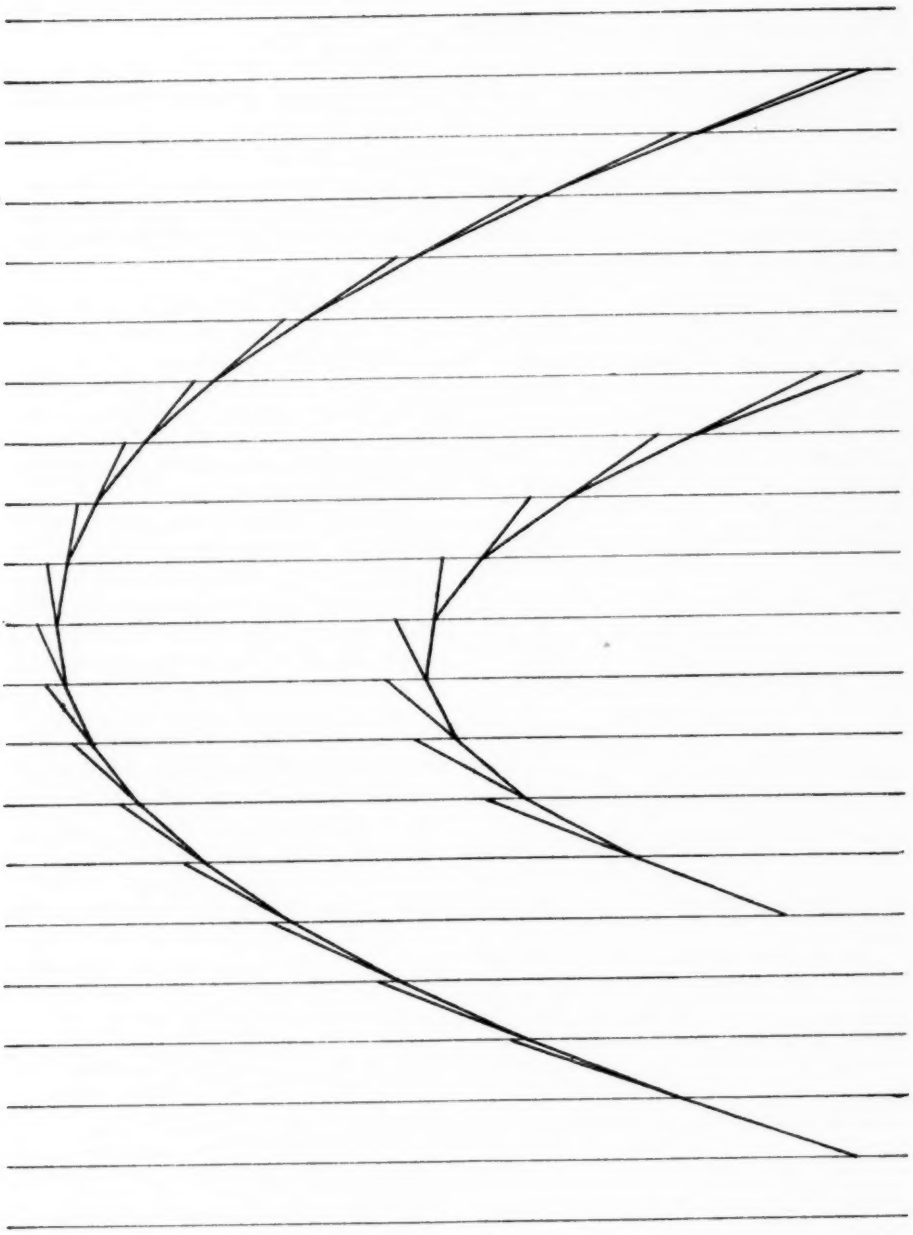

*Figure 2*

# *Old wine in new bottles*

*Contributed by Elizabeth Brown, Arsenal High School, Pittsburgh, Pennsylvania*

Here is an approach to the old problem: Given a three-quart jar, a five-quart jar, and a container holding at least eight quarts, how can you measure exactly four quarts?

Let $x$ = the number of times the three-quart jar is filled
$y$ = the number of times the five-quart jar is filled

A negative value of $x$ or $y$ will mean emptying into the container instead of filling.

Consider $3x+5y=1$. This has an infinite number of integral solutions of which we find one to be

$$3(+2)+5(-1)=1$$

and multiplying by 4

$$3(+8)+5(-4)=4$$

and in general

$$3(+8-5t)+5(-4+3t)=4.$$

For $t=1$ and 2 we obtain the solutions $x=+3$, $y=-1$ and $x=-2$, $y=-2$. The first solution translated into English means:

*Fill* the three-quart jar, pour it into the five. *Fill* the three again, pour all you can into the five. *Empty* the five into the source. Pour the one quart which remains in the three into the five. *Fill* the three from the source and pour it into the five.

If we consider the co-ordinates $(x_i, y_i)$ to represent the amounts in the five-quart jar and the three-quart jar respectively, we can represent the above solution by the points:

(0, 0), (0, 3), (3, 0), (3, 3), (5, 1),
(0, 1), (1, 0), (1, 3), (4, 0).

Steinhaus in "Mathematical Snapshots" (New York: Oxford University Press, 1950) indicates a geometrical method of solution using a rhomboid billiard table with angles of 60 and 120. By driving the ball along the short side of the table (as shown in Figure 3), we trace out the path given by the above co-ordinates. This consists of eight line-segments—the eight acts that give us the four quarts. If the initial drive had been along the long side of the table there would have been seven acts, which in some ways would be a simpler solution. Moreover, the first solution requires pouring a total of 23 quarts, while the second requires a total of 22.

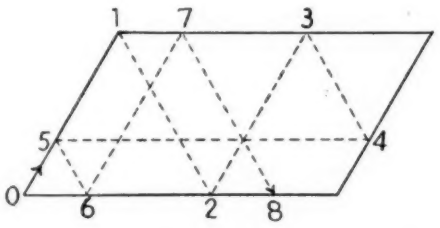


*Figure* 3

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Is it true that the majority of us are born mathematical imbeciles? Why is there this general fear of, and dislike for, mathematics? Is mathematics really so difficult and repelling, or is it the way mathematics is treated and taught by mathematicians that is at fault? If some light can be thrown on these perplexing semantic problems, perhaps we shall face a scientific revolution which might deeply affect our educational system and may even mark the beginning of a new period in standards of evaluation, in which mathematics will take the place which it ought to have. Certainly, there must be something the matter with our epistemologies and "psychologies" if they cannot cope with these problems.—*Alfred Korzybski in* Science and Sanity.

## • REFERENCES FOR MATHEMATICS TEACHERS

*Edited by William L. Schaaf, Department of Education, Brooklyn College, Brooklyn, New York.*

# *Scales of notation*

For a variety of reasons, considerable interest has been shown in recent years in the matter of scales of notation, or the study of systems of numeration having different bases. Prospective teachers of arithmetic, for example, are urged to familiarize themselves with numbers expressed on a scale other than 10, in order to gain further insight into the structure of systems of numeration in general. Another reason for the attention paid to this subject is the dramatic development of electronic computers, related as it is to the use of the binary system. Further interest in the binary system derives from certain mathematical recreations, notably the game of Nim. A certain amount of enthusiasm is also expressed from time to time for the possibilities of using the base 8 or 12 as a more practical base than 10 for purposes of everyday computation. In this connection it is interesting to note that the Duodecimal Society of America was established almost ten years ago.

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# *Reviews and evaluations*

*Edited by Roderick C. McLennan, Arlington Heights Township High School, Arlington Heights, Illinois, and Richard D. Crumley, University of South Carolina, Columbia, South Carolina.*

## BOOKS

SECONDARY

*Algebra Book 2*, C. A. Smith, W. Fred Totten, and Harl R. Douglass, Evanston, Row, Peterson and Company, 1954. Cloth, iii +502 pp., $2.72.

This is one of the rare textbooks that a pupil can be "turned loose" in without disaster. It is clear, literate, and mathematical. All the writing is beautifully done, fundamental meanings are kept clearly before the reader, the exercises are excellently graded, and coming difficulties are so well anticipated that I should think they would frequently not even materialize. The discussion of the fundamental operations with signed numbers epitomizes the superb quality of the writing. Rarely indeed is so fine a balance between clarity and mathematical sense achieved.

The format is excellent. Pages are not broken up into confusing bits by excessive use of bold-face type, italics, etc. The book lies flat when open.

A very good class might need more exercises and more difficult ones. This is the only criticism of the book and it could easily be obviated by use of an exercise book. Most classes will not need more exercises.

The conservative teacher of long experience and fixed ideas is advised not to be repelled by the "comic" drawings and the use of color. They can't hurt the good pupil and will help the slow one. They are so well done that most of them really clarify the text. The color is particularly helpful in the initial discussion of negative numbers.—*Jackson B. Adkins, The Phillips Exeter Academy, Exeter, New Hampshire.*

*Algebra Notes*, William J. Hazard, New York, Vantage Press, Inc., 1952. Cloth, 1–198 pp., $2.95.

The reviewer finds very little in this book that is worthy of recommendation. It has many misleading statements and many "explanations" that are not explanations. There are also many evidences of a lack of understanding on the part of the author of the topics he is trying to explain. Even if the book did not have these shortcomings, it is difficult to see how a worthwhile use could be made of such a book. Any good text in elementary mathematics does the same thing and provides practice exercises in addition.—*William W. Mitchell, Jr., Phoenix College, Phoenix, Arizona.*

*Arithmetic for Today 7*, Thomas J. Durrell, Adaline P. Hagaman, and James H. Smith, Columbus, Charles E. Merrill Books, 1954. Cloth, ii+316 pp., $0.99 net, $1.32 list.

*Arithmetic for Today 8*, Thomas J. Durrell, Adaline P. Hagaman, and James H. Smith, Columbus, Charles E. Merrill Books, 1954. Cloth, ii+316 pp., $0.99 net, $1.32 list.

The topics usually covered in seventh-grade arithmetic and in eighth-grade arithmetic are covered in these two books. A feature that will appeal to teachers is the Enrichment Section of about fifty-five pages at the end of each book. A note at the bottom of each page of enrichment material indicates where it fits into the work covered in the main part of the book. In the interests of economy the pages are smaller and fewer than in many other textbooks. This means that each page presents the pupil with more than the usual amount of reading. The Teacher's Manual gives help and suggestions for each lesson. A feature of the lesson helps entitled *Before Using the Book* is especially good.—*Elizabeth Roudebush, Director of Mathematics, Seattle Public Schools, Seattle, Washington.*

*Die Elemente der Mathematik*, Georg Wolff *et al.*, Hannover, Hermann Schrodel Verlag, 1952–53. Vorstufe, *Rechnen*, Heft 1, paper, 119 pp., DM 2.80; Heft 2, paper, 80 pp., DM 2.80; Heft 3, paper, 80 pp., DM 2.80. Mittelstufe, Band 1, *Arithmetik und Algebra*, cloth, 303 pp., DM 6.30. Band 2, *Geometrie und Ebene Trigonometrie*, cloth, 300 pp., DM 6.20. Oberstufe, Band 3, *Arithmetik, Algebra, und Analysis*, cloth, 352 pp., DM 6.80. Band 4, *Analytische Geometrie, Vektorrechnung, Darstellende und Projektive Geometrie, Spharische Trigonometrie*, cloth, 288 pp., DM 7.40.

This series of seven textbooks, edited by Georg Wolff, an outstanding mathematical educator of Germany, and contributor to The National Council of Teachers of Mathematics' yearbooks, comprises the materials of instruction in the lower, middle, and upper schools of the German Gymnasium, corresponding to our intermediate, junior high, and senior high

school. The three volumes on arithmetic (*Rechnen*) cover the entire theory of counting and measurement, operations with whole number, common fractions, and decimal fractions, per cent, and proportion. Concrete geometry and graphical representation are included. The two volumes for the middle grades (Mittelstufe) give a complete treatment of number systems and the axioms governing them, algebra through quadratics, plane and solid geometry, and plane trigonometry. The upper grades (Oberstufe) include a complete treatment of the analytic geometry of conic sections, theory of equations, differential and integral calculus, vector analysis, descriptive and projective geometry, and spherical trigonometry.

The material covers all that is usually taught in American colleges through the sophomore year. The unique presentation of many of the topics, including historical developments, gives a wealth of material to supplement treatments in American textbooks. The teacher in the United States of America will find the comparison of what is taught in the German Gymnasium with the typical American program a real shock, but we must remember that the Gymnasium is highly selective in its student body (upper 10 per cent of intelligence) and exceptionally rigorous in its academic requirements.—*Howard F. Fehr, Teachers College, Columbia University, New York.*

*Mathematics for Everyday Living*, Adele Leonhardy and Vivian B. Ely, New York, D. Van Nostrand Company, Inc., 1954. iii +470 pp., $2.96.

In recent years a number of textbooks have been published whose audience is those students in grades nine through twelve who are not able or not willing to study technical mathematics. Representative of this group is the present textbook, which treats the "mathematical aspects of the problems of the consumer" and provides an opportunity for a student to improve his arithmetic skills within the context of those problems.

The authors are to be commended for the carefulness and simplicity with which they explain economic problems. There is a wealth of up-to-date, accurate, and very well written problem material to illustrate aspects of the problems of a consumer in our society. Many teachers will object to the separation of the development of a mathematical idea from exercises that involve this idea. Such a style gives the impression that the mechanics of solving problems is the sole objective of mathematics teaching.—*Myron F. Rosskopf, Teachers College, Columbia University, New York.*

*Professional Opportunities in Mathematics*, Report of a Committee of the Mathematical Association of America, Buffalo (Harry M. Gehman, Sec.-Treas., c/o Univ. of Buffalo), Second edition, 1954. Paper, 24 pp., single copies $0.25, orders of ten or more $0.10 each.

More than 10,000 copies of the first (1951) edition have been sold. The report was prepared for undergraduate college students, but is of exceptional value to the high-school teacher or counselor.

The material describes positions available, estimates salaries, and states the preparation required. Five major divisions are covered: the teacher of mathematics, opportunities in mathematical and applied statistics, the mathematician in industry, mathematicians in government, and opportunities in the actuarial profession. There are selected references giving additional information.

With the extremely moderate price, it would seem that no school or college could afford not to have several copies, for the information is probably the most authentic available.—*Cecil B. Read, University of Wichita, Wichita, Kansas.*

*Solid Geometry*, Virgil S. Mallory and Chauncey W. Oakley, Chicago, Benj. H. Sanborn & Co., 1954. Cloth, v+252 pp., $2.64.

This book is the second in the series of geometry books by the authors.

One should be sure the students receive a good understanding of the development of how figures in three-space are drawn and the representation of the principal planes of reference which appear early in the book.

The order of the theorems has been altered somewhat. The proofs of some of the theorems have been anticipated by previous exercises. This facilitates the explanation of many of the proofs.

The authors have made every effort to make the book as readable as possible. They have used both bold face and italicized print to emphasize the important material that is necessary for a good understanding of the subject matter.

The book has numerous exercises and completed and incompleted proofs, which allow the individual class to make up the type of course it wishes to pursue.

At the end of each chapter the authors have provided a summary which helps the student organize his thinking concerning the chapter. Following the summary, there are review exercises and a final test on the chapter.—*Edward Spacapan, Arlington High School, Arlington Heights, Illinois.*

COLLEGE

*Analytic Geometry*, Gordon Fuller, New York, Addison-Wesley Publishing Company, Inc., 1954. Cloth, v+205 pp., $3.85.

This text was planned primarily as a preparation for calculus. A few of the usual topics are not included, while certain others are treated with brevity. For example, much of the geometry of circles is omitted and the normal form of the straight line receives only incidental mention.

The first chapter is devoted to functions and graphs to further the student's experience on those topics. The transformation of co-ordinates concept is introduced early, preceding the study of conics. A chapter is given to the use of derivatives of polynomials and of negative powers of a variable with applications made to the construction of graphs and the solution of maxima and minima problems. The chapter on curve-fitting uses the method of least squares in fitting a straight line to empirical data. In the chapters on solid analytic geometry vectors are introduced and applied in the study of planes and lines.

The format of the book is excellent. The diagrams are well labeled and large enough to be effective. Exercises are well chosen and answers to the odd-numbered problems are included.

The author is to be complimented on a text which is sound though it departs somewhat from the traditional order of topics. The reviewer is favorably impressed by the subject matter and its organization and feels that it is particularly adapted to a well-prepared group of students. For a class of average ability and preparation it would seem necessary to make several omissions, some of which are suggested by the author.—*C. H. Lindahl, Iowa State College, Ames, Iowa.*

*Basic Skills in Mathematics*, H. Vernon Price and Lloyd A. Knowler, Boston, Ginn and Company, 1952. Cloth, viii+249 pp., $3.25.

Designed for a college mathematics course for students specializing in other fields, this book begins with a review of the basic concepts and skills of arithmetic and geometry. The remaining chapters deal with selected concepts and skills of high-school algebra and geometry, of finance, and of statistics.

As the title indicates, emphasis is placed on skills rather than on meaningful concepts or on ways of thinking. The reviewer feels that this rules out the use of the book for a course in mathematics for general education. The best use would be for those students who are found deficient in mathematics but who need the basic skills of mathematics in connection with some other area of study. Even if used this way, it would be necessary for the instructor to provide for more adequate development of the concepts included and for additional applications of the skills.—*Richard D. Crumley, Assistant Professor of Education, University of South Carolina, Columbia, South Carolina.*

*Calculus*, George B. Thomas, Jr., Cambridge, Addison-Wesley, 1953. Cloth, 1–614 pp., $6.50.

This text is designed for the early introduction of integral calculus. Concepts of limits and continuity appear with some emphasis on the use of inequalities. The topic of infinite series is introduced nicely with some of the basic ideas of sequences. The introduction of the method of variation of parameters in the brief treatment of differential equations may be unusual in an elementary calculus text but may have merit.

Problem lists seem to be adequate to provide for ample drill. Answers to most of the odd-numbered problems appear in the back of the text. There is an appendix of formulas from elementary mathematics. Drawings included in the text are exceptionally instructive.

This book may well fulfill the need for those who desire that a text have a little more than the minimum amount of material. In accord with the author's suggestion several of the topics, such as vector analysis and line integral, might be postponed until a later course.—*J. Ray Hanna, University of Wichita, Wichita, Kansas.*

*Calculus—An Introduction to Analysis, and a Tool for the Sciences*, Gaylord M. Merriman, New York, Henry Holt and Company, 1954. Cloth, vii+625 pp., $6.50.

This book presents in sequence three main functional analyzers: the differential, the anti-differential and the definite integral in the first 250 pages. The operational symbol $d^{-1}$ is used to denote anti-differentiation. The well-chosen illustrative examples and problems follow the line of traditional calculus and give practical value to the book. Each chapter contains a summary and list of miscellaneous exercises. The writer has included many ideas and exercises that will stimulate thinking. The material in this book is sufficient to serve the standard four- or five-hour year course. The format and typography are pleasing. The author's choice of figures and terminology is conventional. The usual list of integrals, review formulas, tables and index appear in the final thirty-three pages.—*Orlando C. Kreider, Iowa State College, Ames, Iowa.*

*Differential Equations with Applications*, Herman Betz, Paul B. Burchman, and George M. Ewing, New York, Harper and Brothers, 1954. v+310 pp., $4.50.

This is a very nice introductory text. The first six chapters, comprising approximately the first half of the book, consist of a good treatment of the material on first order and linear equations which is standard in essentially all introductions to differential equations. The applications are varied and are presented in an interesting manner. The second half of the book considers slightly more advanced techniques such as power series and Fourier solutions (with boundary values). A novel feature for a book of this kind is an elementary and clear introduction to the use of Laplace transforms. There are numerous worked-out examples, sufficiently many exercises, and a few references are given at the end of some of the chapters.—*M. L. Curtis, Northwestern University, Evanston, Illinois.*

*Elementary Theory of Numbers*, Harriet Griffin, New York, McGraw-Hill Book Company, Inc., 1954. Cloth, ix+203 pp., $5.

In a fairly short beginning are included the postulates, definitions, and methods of proof

which make up the basic structure of the theory of numbers. This approach followed by a clear discussion of each new concept which is developed gives this book an extra readability for students of mathematics as well as makes it available to a layman interested in the theory of numbers who has one sound algebra course in his background. Although it is primarily intended as a text for an undergraduate or graduate course, this book might be used to an advantage by teachers of secondary mathematics who are looking for challenging and understandable reading for exceptional students on that level, particularly in its earlier chapters.

The classical theory of numbers, the linear Diophantine equation, divisibility, congruences, indices, quadratic residues, partitions, famous theorems, and problems connected with the subject are included with a chapter on polynomials, integral domains and fields. The material is grouped in such a way, and with sufficient historical background, to give it unity as well as logic in its development. Typical of this is the seventh chapter, a development of the theory and theorems connected with "On Belonging to an Exponent."

This is one of the International Series in Pure and Applied Mathematics presented by this publisher.—*Virginia Carlton, Wesleyan College, Macon, Georgia.*

*Elements of Statistics*, H. C. Fryer, New York, John Wiley & Sons, Inc., 1954. viii+262 pp., $4.75.

This is a textbook in descriptive statistics, stressing the concepts and uses of distribution, correlation, and sampling. Mathematical formulas are presented and used, but the derivations of most of the formulas are not given. A short chapter on probability provides that mathematics needed, in addition to high-school algebra, to understand the text material. An unusual inclusion is an approximate G-test for use with small samples. The text is well adapted to a college introductory course for non-mathematics majors.—*Howard F. Fehr, Teachers College, Columbia University, New York.*

*A First Course in Ordinary Differential Equations*, Rudolph E. Langer. New York, John Wiley & Sons, Inc., 1954. Cloth, v+249 pp., $4.50.

Here is a text on differential equations which is different. There are no tables of formulas or lists of rules. Instead of the usual grab bag collection of unrelated tricks, the author presents us with a beautifully logical exposition of the elementary theory of this subject. The emphasis is properly placed on first and second order equations, and these are treated geometrically and graphically as well as analytically. The specific techniques described are kept to an essential minimum. The end result is a unity of treatment which is seldom found in textbooks on this subject.

This book is also to be recommended because of its readability. The author has managed to clearly define all new terms and to use language correctly without destroying the simplicity of the exposition. The reader, who is assumed to have had a year of calculus, can read straight through the text if the first reading includes the illustrative examples. The applications of differential equations in physics, chemistry, and engineering are carefully discussed and sufficient in number without being overwhelming.

Langer's *Differential Equations* is a fine addition to the reference shelf of both student and teacher, as well as a good text for classroom use. —*Augusta Schurrer, Iowa State Teachers College, Cedar Falls, Iowa.*

*Introduction to College Mathematics*, 2d. ed., Carroll V. Newsom and Howard Eves, New York, Prentice-Hall, Inc., 1954. Cloth, 8+408 pp., $5.75.

Those who have used the first edition of this book will find this revision equally suited to their needs. Others will find it quite adequate for the purpose for which it is designed. That purpose is to furnish a readable textbook for the fairly capable student who plans to take no more than one course in college mathematics. Although little mathematical background is a necessity, two years of high-school mathematics will be found most helpful. The most noticeable changes are the omission of Chapter 1, "The Nature of Mathematics," and the inclusion of a chapter on statistics and one on the calculus. This book will give the student a well-rounded introduction to the most interesting parts of algebra, trigonometry, analytic geometry, and the calculus with many applications of each. Some of the theory is developed through examples and problems, and the student is urged to do a great many of them. There are many historical notes, an appendix containing needed tables, and the answers for about half the exercises. The print is clear and the cover attractive.—*Frank C. Gentry, University of New Mexico, Albuquerque, New Mexico.*

*Introductory College Mathematics*, Chester George Jaeger and Harold Maile Bacon, New York, Harper & Brothers, 1954. Cloth, v+382 pp., $4.75.

This is a college text presenting in a unified manner most of the material usually found in the separate courses in college algebra, trigonometry, and analytic geometry. It includes more of the calculus than other books of this type. The organization of material is refreshingly different; for example, the student uses the integral calculus to find volumes of revolution before considering ordinary logarithms. The function concept is used throughout the text.

The text provides sufficient background for further work in mathematics, but it is also suitable for those students who need more than usually presented in the compartmentalized courses but whose program permits only one year of mathematics.

In general the book is well written. However, in relaxing rigor in favor of lucidity, the authors make the following statement in introducing determinants, "A device used to solve simultaneous linear equations is called a determinant."—*Arnold Wendt, Western Illinois State College, Macomb, Illinois.*

*Slide Rule—A Textbook for Classroom and Self Instruction*, C. L. Johnson, Dubuque, Iowa, Wm. C. Brown Company, 1953. Paper, iii +86 pp., $2.

The book is primarily for those who have had mathematics courses beyond trigonometry. The layout is pleasing and illustrated examples showing slide rule settings for readings and solutions are good. Included with the examples are detailed steps of solution and methods of determining decimal points. The reviewer wonders if there may not be too much discussion of the location of decimal points by involved processes rather than by simpler approximation methods. It appears that the book would be most useful in a classroom situation where it would serve well. The student might have some difficulties in using the book as a self-teaching guide.—*George Larsen, Arlington Heights High School, Arlington Heights, Illinois.*

*Theory of Equations*, Cyrus Colton MacDuffee, New York, John Wiley & Sons, Inc., 1954. Cloth, v+120 pp., $3.75.

This text is superior to many of its type in at least three respects: (1) Topics which are usually covered in first-year college courses, such as systems of linear equations, are presented in such a way that the student will be aware of insights and understandings he missed as a freshman. (2) The student will benefit in encountering the "arbitrarily small epsilon" notation of his calculus course in several places such as in proofs of Budan's and Sturm's theorems. (3) Several modern algebra concepts, such as ring and automorphism, are woven in skillfully and naturally.

The omission of determinants and matrices is a disadvantage in the instance where an additional course covering these topics is not taken.—*Kenneth W. Wegner, Carleton College, Northfield, Minnesota.*

*Understanding College Algebra*, Edwin R. Smith, Samuel Selby, and Murray Kleiman, New York, The Dryden Press, 1954. Cloth, v+573 pp., $3.50.

The subject of college algebra is thoroughly motivated in this text through the use of historical accounts and well-chosen applications. The choice of topics is not unusual, although a few of them are discussed in a little more detail than is customary.

Review topics of such items as operations with signed numbers, fundamental operations, etc. are treated adequately. A listing of common student errors is a desirable feature of the book. Problem lists seem to contain an ample selection so that alternate assignments could be made. Answers are given to most of the odd-numbered problems. There is an appendix containing material on the derivation of $\pi$, the irrationality of $\sqrt{2}$, and De Moivre's theorem. Another appendix contains the usual tables.

In defining negative exponents, page 73, the restriction $a \neq 0$ has been omitted. Although some might question the necessity for discussing the theory of common logarithms and the theory of logarithms having bases other than ten, separately, the plan probably has merit so far as student understanding is concerned.

This book should serve the needs of college freshmen who present three semesters of high-school algebra for college entrance. The text is certainly a refreshing addition to the subject of college algebra.—*J. Ray Hanna, University of Wichita, Wichita 14, Kansas.*

MISCELLANEOUS

*C R C Standard Mathematical Tables* (Tenth edition) (formerly Mathematical Tables from Handbook of Chemistry and Physics), Chas. D. Hodgman, Editor, Cleveland, Chemical Rubber Publishing Co., 1954. ix +441 pp. $3.

This collection of tables is well known; certain additions and changes have been made in this edition: New tables: four place common logarithms of numbers between 0 and 1 (expressed as negative numbers); squares and products of the sine and cosine; radians to degrees, minutes, and seconds and vice versa; the first 101 powers of 2; $t$, $F$, and chi square for statistics; additional elliptic integrals; sine, cosine, and exponential integrals; Laplace transforms; CSO mortality tables and commutation columns, 2½ per cent. The range of values of hyperbolic and exponential functions has been enlarged and extended; the integral tables have been enlarged. There is new material on vector analysis and additional material on trigonometry.

Some would prefer an extension of the range of values for trigonometric functions in radian measure; others might like a brief edition containing no material beyond sophomore college level—but everyone cannot be pleased. This reviewer would like to see mention of alternative definitions of the inverse trigonometric functions; he does not agree that $a^0 = 1$ always (even for $a = 0$) as stated on page 413. A minor typographical error present in earlier editions remains (page 422).—*Cecil B. Read, University of Wichita, Wichita, Kansas.*

*MATHeMAGIC, Magic, Puzzles and Games with Numbers*, Royal Vale Heath, New York, Dover Publications, Inc., 1953. Paper, 126 pp., $1.

Magic mathematical effects which were once claimed by mathematical geniuses and prodigies are now available to everyone in this book. You select your own digits, the author tells you what

to do with them and predicts the results of your calculations. Besides the "magic," the book contains unusual and interesting treatments of numbers called symphonies. Two of the chapters entitled "Easy ways to multiply," and "Arithmeticklish" would be usable for high-school teachers. This book has value in both teaching and entertainment of self and friends.—*Lola May, New Trier Township High School, Winnetka, Illinois.*

*Mathematics, Minus and Plus*, Leeds K. Field, New York, Pageant Press, 1953. Cloth, 15 pp., $2.

On the opening page of his treatise, Mr. Field throws out the time-honored conception of pi and goes on to "prove" that pi is equal to 3.0. Obviously this is a strictly hypothetical work, of value primarily to those interested in a developing, theoretical science of mathematics. It offers little to the strictly practical-minded person but is an extremely intriguing approach to the ancient problems of squaring the circle, duplicating the cube, and other related work. An attempt is made, with considerable success, to emphasize the importance of the equilateral triangle in modern mathematics.

To one who is willing to open his mind to new ideas in much the same way as when studying non-Euclidean geometry or a number system other than decimal, a fascinating evening may be profitably spent.—*Robert V. Belding, Arsenal Technical Schools, Indianapolis, Indiana.*

## EQUIPMENT

*Conic Sections (7570); Hyperbolic Sections (7571); Dissectible Pyramid (7574); Trisectible Prism (7575); Binomial Cube (7591)*; W. M. Welch Scientific Company, 1515 Sedgwick Street, Chicago 10, Illinois. Plastic models, various sizes and prices.

*Description:* These plastic models of certain solid figures are just five of a larger set covering topics in secondary school and college mathematics. In general the plastic is transparent and the model is non-dissectible, but when appropriate, colored plastic and removable sections are used. The numbers after the titles are the company's order numbers. They will be used in the following descriptions: 7570: The cone is 270 mm. high (10″) and 130 mm. in diameter (5″). Three sections are shown inside: a parabola in blue, a hyperbola in red, and an ellipse in yellow. The price is $14.50.

*7571:* The two nappes of a right circular cone 210 mm. high (8″), and 110 mm. in diameter (4″) are cut by a colored plane parallel to the axis, thus showing the two branches of the hyperbola. The price is $15.

*7574:* A square pyramid 200 mm. high (8″) with a side of the base equal to 100 mm. (4″) has a removable piece at the top, a smaller pyramid 100 mm. (4″) high, leaving a frustrum of a pyramid. The price is $10.

*7575:* A right triangular, transparent prism contains three pyramids, red, blue and green, illustrating the usual figure for proving the volume of a pyramid. The whole model is 190 mm. high (7½″), has an equilateral-triangle base 100 mm. (4″) on a side, and costs $25.50.

*7591:* A transparent 100 mm. (4″) cube contains the dissection blocks needed to demonstrate $(a-b)^3$, where $a$ is 60 mm. (2½″) and $b$ is 40 mm. (1½″). The $a^3$ block is red, the $a^2b$ blocks blue, the $ab^2$ blocks yellow, and the $b^3$ block green. The cost is $17.50.

*Evaluation:* These models are as useful and beautiful as any teacher can ever wish. They are large enough for use in a fairly large group, sturdy enough to stand constant and hard use, and designed (with respect to both component parts and use of color) with real teaching situations in mind. Whether they are bought for teacher demonstration, pupil manipulation and measurement, or museum display, they will be more than satisfactory. At first the prices will seem high, but get the models into your hands, try them in the classes you teach, listen to the resulting discussion and you will be convinced. Plastic is very necessary here, and it is a fairly expensive material. Also, there is no substitute for good workmanship, and these models certainly have it.—*Henry W. Syer, School of Education, Boston University, Boston, Massachusetts.*

## FILMSTRIPS

*Visualizing Number Pictographs (578); Bar Graphs (579); Line Graphs (580); Circle Graphs (581); Graphs in Daily Life (582)*; Educational Projections, Inc., 10 East 40th Street, New York 16, New York. Filmstrips: 35 mm., color; each 26 frames; $21.50 (for complete kit described below).

*Description:* These five filmstrips come packed in a box with other material to teach about graphs: 8 charts to graph on (each 24″ ×36″), and 50 protractors, with 360° on one side and a 100-point, percentage scale on the other (4″ in diam.). Subtracting the $4, which extra graphs cost, and $2.50 for protractors, we get down to a very sensible $15 for the five filmstrips.

The eight charts are as follows: 1 circular, with one circle; 1 circular, with two circles; 2 vertical rectangular, with grid 30″×16″, and place for title and scale; 4 horizontal rectangular, with grid 30″×16″, 2 with title and scale places, 2 with title places only. The subjects of the filmstrips may be inferred from their titles.

*Evaluation of (578):* A technique of showing incomplete pictographs with the question, "What is missing from the pictograph?" is excellent and leads to good discussion. The presence of tables of data along with graphs so that pupils may compare the two and learn interpretation is also very useful. The filmstrip ends with a comparison of a bar graph, a line graph, and a circle graph. This is good preparation for the other

topics to come, but in general the preparation and review sequences give the appearance of too much overlapping of material, and not enough of the material is concentrated on the subject at hand.

*Evaluation of (579)*: The chief point of this filmstrip is the comparison of bar graphs and pictographs. The over-all appearance of the art work in the illustrations is amateurish and sometimes even sloppy. It is not the carefree or naive quality which good artists sometimes give to their work, but just poor workmanship. It does not interfere with the usefulness of the concepts presented, however.

*Evaluation of (580)*: Some teachers may feel that the statement "The main purpose of any line graph is to show a trend—what happens over a period of time" is too narrow and restricting. There will also be discussion of the actual encouragement of vertical scales which do not start with zero. The series contradicts itself slightly by sometimes prohibiting the reading of values between plotted points, and sometimes allowing it. In general, it must be pointed out that the criticisms being found are far outweighed by the excellent helpfulness of the pictures; for example, questions concerning the interpretation of graphs are pertinent and require response, not passive viewing.

*Evaluation of (581)*: A good filmstrip, helpful to the teacher.

*Evaluation of (582)*: Again, we have a series of excellent graphs for interpretation. The summary of uses of the four types is clever, but not true. Would all teachers assign just one function to them as follows: Pictograph—count; Bar graph—compare; Line graph—trend; and Circle graph—relationships?

*Evaluation of other material:* The idea of buying a kit of materials to teach graphs may appeal to some teachers, and it is certainly useful from the merchandising standpoint for the producer. If it is easy to buy whatever component parts a teacher wishes, the idea is to be commended; if only the complete package is offered, the idea is a disservice to education. The 360° protractors are very useful, usually difficult to find, and especially ingenious in having the 100-point scale on the back. Many teachers would like to have these. On the other hand, there seems to be no excuse in the world for the charts. What teacher can, or should, spend fifty cents for a piece of paper with a circle printed on it? Even a sturdy, attractive piece of paper. Even the rectangular graphs are not needed in the luxurious form they are offered. Making special plates to print horizontal and vertical graphs, and also to print some with the words "TITLE" or "SCALE" on them, show some ignorance of present-day school budgets. The filmstrips will be useful to all teachers who buy them, the protractors to those who need that special instrument, but the charts should be eschewed.—*Henry W. Syer, School of Education, Boston University, Boston, Massachusetts.*

## PAMPHLETS

*Orienting Ourselves in Our World with Globes*, The George F. Cram Company, Inc., 730 East Washington Street, Indianapolis 7, Indiana. Pamphlet, 6″×9″, 15 pp., free.

*Description:* It is the purpose of this pamphlet to emphasize the fact that visualizing our world correctly is vital to understanding it. Globes are the best devices available for an accurate representation of the earth. Although educationally the value of globes has been recognized for years, globes have not been used to their fullest extent. To understand the replica of the earth, students must be familiar with the scale of the globe and must know where countries or cities are in relation to one another. Meridians, parallels of latitude, parallels as iso-sun lines, meridians as iso-sun time lines, and great circle routes are discussed extensively. The concepts of the world essential for beginners are illustrated in reference to Cram's twelve-inch simplified globe.

*Evaluation:* Many facts are given in this article concerning the movement of the earth about the sun and what to expect with regard to temperature at various places on the earth during different parts of the year. It contains a good summary of material that can be used in a unit on mapping for a geometry class. Despite the brief treatment of the material, it is presented with remarkable simplicity and clarity.—*Margaret Linster, St. Louis Park, Minnesota.*

*Teaching the Skills Necessary for Getting Information from Graphic Sources as a Background for Critical Thinking*, The George F. Cram Company, Inc., 730 East Washington Street, Indianapolis 7, Indiana. Pamphlet, 6″×9″, 16 pp., free.

*Description:* Mr. Hobson in his article emphasizes that if education is to promote democracy, it must inculcate the attitudes and skills involved in critical thinking; it must teach pupils to solve the social problems that threaten the development of a truly democratic society. The purposes of this article are: first, to examine briefly the crucial social issues to which critical thinking must be applied; second, to enumerate the attitudes and techniques underlying critical thinking in the social field; and third, to list suggestive classroom activities which may be carried on by teachers and pupils for learning how to get facts from graphic presentations.

*Evaluation:* All teachers need help in planning and conducting the activities necessary to teach critical thinking, and only a very meager number have had any guidance along these lines. This article should focus the teacher's attention upon developing critical thinking among students. Illustrations of the standard types of reasoning problems are given. Some of the activities suggested could well be used in a mathematics class.—*Margaret Linster, St. Louis Park, Minnesota.*

## • TIPS FOR BEGINNERS

*Edited by Francis G. Lankford, Jr., University of Virginia, Charlottesville, Virginia.*

# *Solving percentage problems by the equation method*

*By Marvin C. Volpel, Maryland State Teachers College, Towson, Maryland.*

*The author calls attention to the fact that many seventh-grade problems are of the general type* a b c *and used this fact to develop a method for teaching percentage problems.*

Presenting problems using the three cases of per cent is often as much a problem for the teacher as the solution of them is for the student. Several years ago when the author was a seventh-grade pupil in a rural school in Michigan he was taught to solve percentage problems by the formula method. He was asked to memorize the formula $b \times r = p$ and its counterparts, $b = p \div r$ and $r = p \div b$. The solution of a problem in percentage meant the selection of one of the formulas and then the manipulation of terms until a reasonable answer was obtained. The method for its solution had neither rhyme nor reason and left the author no better informed regarding the meaning of "per cent." It was always difficult to select the proper number for the base and doubly difficult to distinguish between "per cent" and "percentage." It was not until the author was in high school that he learned that per cent meant "by the hundred" and that percentage was not the size of the per cent but a specific part of the original amount.

Dr. Raleigh Schorling of the School of Education, University of Michigan, was one of the early proponents for the teaching of simple algebra in the junior high school. He believed that children at the seventh-grade level could be taught to solve simple equations of the type $3n = 18$, $.35n = 70$, $\frac{3}{4}n = 9$, and $.08n = 32$, as only a knowledge of the division axiom would be needed. Dr. Schorling made a strong appeal for teaching percentage problems by this method. The writer used the equation method in presenting percentage problems to his pupils in the junior high school and immediately met with such marked success that he has been using this form for presentation ever since and recommends it to others.

One of the biggest tasks in this type of attack is to develop the pupils' ability to translate the verbal statements into symbols. This is made easier if the problem is written on one line across the page or blackboard. Also, it is helpful if the pupils recognize the following: (1) that the word "is" divides the statement into two parts and means "equals," and (2) that the word "of" is synonymous to the word "times."

*Contributions dealing with the problems faced by the beginning teacher of mathematics will be welcomed by the editor of "Tips for Beginners."*

Case I is the easiest of the three types of problems. Its solution involves only one operation, namely multiplication. A sample problem is:

What is 12% of \$40?

*Solution:*

$$n = .12 \times 40$$

$$n = \$4.80$$

Other problems of this type are: (1) Find 45% of \$65. (2) What is the tax on a \$45.00 coat if the sales tax is 3%? (3) Mr. Jones spends 17% of his yearly income for rent. If his yearly income is \$6700.00, how much does he spend for rent?

In Case II problems, one is asked to find a per cent. For example: What per cent of 25 is 18?

*Solution:*

Let $n$ represent the decimal equivalent of the per cent.

Then

$$n \times 25 = 18.$$

This is the same as $25 \times n = 18$ or $25n = 18$. Dividing both sides of the equation by 25

$$n = .72 \text{ or } 72\%.$$

Another problem of this type is: If a man earns \$500.00 per month and spends \$150.00 for clothes, what per cent of his income does he spend for clothes?

*Solution:*

Let $n$ represent the decimal equivalent of the per cent.

Then

$$n \times 500 = 150 \text{ or } 500n = 150.$$

Dividing both sides of the equation by 500

$$n = .30 \text{ or } 30\%.$$

A popular problem of this type is this one: If a team won 96 out of 154 games played, what per cent of the games played did it win?

*Solution:*

$$n \times 154 = 96 \quad \text{or} \quad 154n = 96$$

$$n = .623 \quad \text{or} \quad 62.3\%.$$

In Case III problems, one is asked to find the whole amount when a part is known. A sample problem is:

3% of what number is 24?

*Solution:*

$$.03 \times n = 24 \quad \text{or} \quad .03n = 24$$

$$n = 800.$$

Another problem of this type is: If one contributes 10% of his earnings to the church, what must he earn per year in order to be able to contribute \$350.00 to the church?

*Solution:*

$$.10 \times n = 350 \quad \text{or} \quad .10n = 350$$

$$n = \$3500.$$

It is helpful to translate word statements into equations and equations into word statements when teaching by the equation method. Equations similar to the following could be used:

$$.45n = 990 \quad n = .35 \times 750 \quad 72n = 96.$$

Many of the arithmetic problems solved in the sixth and seventh grades are of the general form $a \times b = c$, and students should be familiar with the relationships between these variables. The students have learned that if two numbers are multiplied to obtain a certain product, then that product divided by either one of the multipliers will give the other. Since $6 \times 7 = 42$, then 42 divided by 6 equals 7. Students are familiar with the fact that the cost of an article when multiplied by the number of articles gives the total cost ($n \times p = c$), and that, when the cost is known and one is to find either the cost per article or the number of articles, the answer is obtained by division. Children in the elementary school have also had experience solving problems

involving the distance, rate, and time relationship. Since $r \times t = d$ then when $d$ is divided by either $r$ or $t$ we obtain the value of the other variable. Another familiar problem involving this $a \times b = c$ relationship is the problem of the area of a rectangle. Pupils learn that the area of a rectangle is the product of its two dimensions and is expressed in square units. Since $l \times w = A$, then when the area and width are known the length is found by division, $l = A \div w$. We must not lose sight of the fact that both sides of the equation are being divided by the same number.

Most courses of study for junior high-school mathematics include work on "interest." Although another variable is introduced in this relationship, the problems resolve themselves into one of the three cases of percentage. Instead of teaching four different formulas for use in solving "interest" problems the writer recommends the use of only one formula, $p \times r \times t = I$. When one is to solve for either $p$, $r$, or $t$, the proper substitutions and multiplications reduce the equation to the form of $a \times b = c$.

The equation method for solving percentage problems makes use of a basic relationship which has been established in the arithmetic program of the elementary school. For this reason, the use of the equation provides a simple and straightforward analysis and solution of percentage problems. Furthermore, a large number of junior high-school problems can be fitted to this type of analysis. For these reasons, the writer believes that the equation method is the best approach to the solution of percentage problems.

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# *Your professional dates*

## NCTM convention dates

CHRISTMAS MEETING

December 27–29, 1954
Chase Hotel, St. Louis, Missouri

Jesse Osborn and
Margaret Willdering, local chairmen
Harris Teachers College
5351 Enright Avenue
St. Louis 12, Missouri

ANNUAL MEETING

April 13–16, 1955
Statler Hotel, Boston, Massachusetts

Jackson Adkins, local chairman
Phillips Exeter Academy
Exeter, New Hampshire

JOINT WITH NEA

July 4, 1955
Chicago, Illinois

SUMMER MEETING

August 21–24, 1955
Indiana University, Bloomington, Indiana

Philip Peak, local chairman
Department of Education
Indiana University
Bloomington, Indiana

## Affiliated groups convention dates

October 9, 1954
California Mathematics Council, Southern Section
South Pasadena Junior High School
South Pasadena, California

William H. Glenn, Jr., President
Pasadena City College
Pasadena 4, California

November 5, 1954
Missouri Council of Teachers of Mathematics in conjunction with the Missouri State Teachers Association
Continental Hotel, Kansas City, Missouri

Margaret F. Willdering, President
1283 Mead Drive
St. Louis, Missouri

## • WHAT IS GOING ON IN YOUR SCHOOL?

*Edited by John A. Brown, University of Wisconsin, Madison 6, Wisconsin, and Houston T. Karnes, Louisiana State University, Baton Rouge 3, Louisiana.*

# *A scrapbook in algebra*

*Contributed by Olga A. Tapper, Franklin School, Cedar Rapids, Iowa*

THE ALGEBRA CLASS at Franklin School, Cedar Rapids, Iowa, made an interesting scrapbook as a special class project. Pencil sketches and short biographies of famous mathematicians including Thales, Pythagoras, Plato, Euclid, Archimedes, Descartes, Newton, Napier, Gauss, and Einstein relate a short history of mathematics. Also, the scrapbook contains several magic squares, original cartoons, and puzzles. The class had a program in which they presented original skits on mathematics.

Here are several interesting excerpts from the scrapbook:

"Believe It or Not"

Believe it or not traces of algebra were found as far back as 1700 B.C.

Believe it or not but today a mere beginner in the study of algebra may know more about the subject than the wisest mathematician of antiquity.

Believe it or not but one of the stumbling blocks in the development of the study was the cubic equation. For many years Nicholas Tartaglia, an Italian, worked on the problem, and finally he came upon a solution. This got him into a conflict with Antonio Floridas who had a different solution. It was finally settled in a public debate in 1535. Tartaglia finally proved his method's superiority.

Believe it or not but you learned some very elementary algebra back in grade school. Remember how your teacher would say, "Two and what number make five?" And then think back to some of the first equations that you were asked to solve in Algebra 1. Weren't they like this: $2+X=5$? Isn't this the same as the other one?

Believe it or not but a man named Cardano wrote a book in the year 1545 that was somewhat of an algebra textbook. Although it was very elementary some of his theories are still considered true.

Believe it or not, the study of algebra can be made as absorbing as that of astronomy or literature, and it also offers some exceptional opportunities for mental discipline, since it can be mastered only with persistent attention to detail, and it demands accuracy and concentration in the highest degree.

The scrapbook closes with the following comments from the class:

FRANKLIN SCHOOL

January 16, 1950

"We, the pupils of Algebra, wish to express our appreciation to our school officials for offering this course. It gives you training in quick thinking power, also the power of reasoning. Concentration is one power that is needed in algebra. Although you may not use factoring in your everyday life, you will need concentration, which can be easily developed in this course.

"Just plain arithmetic doesn't go into the subject deep enough. Algebra opens new fields for study and may help you decide if you'd like to go into further training for it.

"Algebra is not all plus and minus signs, neither is it fractions or radicals but it includes spelling. Around Christmas time we had fun unscrambling words pertaining to Christmas. We also had two committees. One for a "Believe It or Not" scrapbook and the other for an Algebra Program. All of you read Bob Ripley's "Believe It or Not" articles and it's things like these that help make algebra interesting. The Algebra Program, which was held at the end of the semester, included an original skit by the program committee, talks on men who contributed to the development of algebra, and stories in song form about algebra to the tune of familiar melodies. All this, along with the help and patience of our teacher, Miss Tapper, made our algebra class one of the nicest we have had. Our thanks to those who made this course possible."

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# *What's new?*

## BOOKS

SECONDARY

*Algebra Book 2*, C. A. Smith, W. Fred Totten, and Harl R. Douglass, Evanston, Row, Peterson and Company, 1954. Cloth, iii +502 pp., $2.72.

*Applied Business Calculation* (Third edition), C. E. Birch and Vivian Wright Kline, New York, Gregg Publishing Company, 1954. Paper, 180 pp., $1.60.

*Die Elemente Der Mathematik, Arithmetik und Algebra Mittelstufe—Band 1; Geometrie und Ebene Trigonometrie Mittelstufe—Band 2*, Georg Wolff, et al., Hannover, Hermann Schroedel Verlag, 1952 & 1953. 303 pp. in Band 1, 300 pp. in Band 2; paper, Band 1, DM 6, 30; Band 2, DM 6, 20.

*Die Elemente Der Mathematik, Rechnen, Vorstufe, Heft* 1, 2, 3, Georg Wolff et al., Hannover, Hermann Schroedel Verlag, 1952–53. 119 pp. in Heft 1, 80 pp. in Heft 2, 80 pp. in Heft 3; paper, Heft 1, DM 2, 80; Heft 2, DM 2, 80; Heft 3, DM 2, 40.

*Emerging Practices in Mathematics Education, Twenty-Second Yearbook*, The National Council of Teachers of Mathematics, Washington, D.C., The National Council of Teachers of Mathematics, 1954. Cloth, v +434 pp., $4.50; members of the Council $3.50.

*Mathematics for the Secondary School—Its Content, and Methods of Teaching and Learning*, William David Reeve, New York, Henry Holt and Company, 1954. Cloth, vii+547 pp., $5.95.

*Slide Rule—A Textbook for Classroom and Self Instruction*, C. L. Johnston, Dubuque, Iowa, Wm. C. Brown Company, 1953. Paper, iii +86 pp., $2.

*Solid Geometry*, Virgil S. Mallory and Chauncey W. Oakley, Chicago, Benj. H. Sanborn & Co., 1954. Cloth, v+252 pp., $2.64.

COLLEGE

*Algebra for College Students* (Revised edition), Jack R. Britton and L. Clifton Snively, New York, Rinehart & Company, Inc., 1954. Cloth, v+537 pp., $4.25.

*Analytic Geometry*, Gordon Fuller, New York, Addison-Wesley Publishing Company, Inc., 1954. Cloth, v+205 pp., $3.85.

*Analytic Geometry* (Second edition), Edward S. Smith, Meyer Salkover, and Howard K. Justice, New York, John Wiley & Sons, Inc., 1954. Cloth, v+306 pp., $4.

*An Analytical Calculus for School and University* (Volume I), E. A. Maxwell, New York, Cambridge at the University Press, 1954. Cloth, ix+165 pp., $2.75.

*An Analytical Calculus for School and University* (Volume II), E. A. Maxwell, New York, Cambridge at the University Press, 1954. Cloth, vii+272 pp., $3.50.

*Calculus* (Third edition), G. E. F. Sherwood and Angus E. Taylor, New York, Prentice-Hall, Inc., 1954. Cloth, v+579 pp., $5.75.

*Calculus—an Introduction to Analysis, and a Tool for the Sciences*, Gaylord M. Merriman, New York, Henry Holt and Company, 1954. Cloth, vii+625 pp., $6.50.

*College Algebra*, H. G. Apostle, New York, Henry Holt and Company, 1954. Cloth. v +432 pp., $4.50.

*College Algebra* (Third edition), Paul K. Rees and Fred W. Sparks, New York, McGraw-Hill Book Company, Inc., 1954. Cloth, v +460 pp., $4.25.

*Die Elemente Der Mathematik Arithmetik, Algebra, und Analysis Oberstufe—Band 3, Analytische Geometrie, Vektorrechnung, Darstellende und Projektive Geometrie, Spharische Trigonometrie Oberstufe—Band 4*, Georg Wolff et al., Hannover, Hermann Schroedel Verlag, 1953. 352 pp. in Band 3, 288 pp. in Band 4; paper, Band 3, DM 6,80; Band 4, DM 7,40.

*Differential Equations with Applications*, Herman Betz, Paul B. Burcham, and George M. Ewing, New York, Harper and Brothers, 1954. Cloth, v+310 pp., $4.50.

*Differential and Integral Calculus* (Fifth edition), Clyde E. Love and Earl D. Rainville, New York, The Macmillan Company, 1954. Cloth, v+526 pp., $5.75.

*Elementary Theory of Numbers*, Harriet Griffin, New York, McGraw-Hill Book Company, Inc., 1954. Cloth, ix+203 pp., $5.

*Elements of Mathematics for Radio, Television and Electronics*, Bernhard Fischer and Herbert Jacobs, New York, The Macmillan Company, 1954. Cloth, v+569 pp., $5.40.

*First Course in Calculus*, Hollis R. Cooley, New York, John Wiley & Sons, Inc., 1954. Cloth, v+643 pp., $6.

*A First Year of College Mathematics* (Second edition), Raymond W. Brink, New York, Appleton-Century-Crofts, Inc., 1954. Cloth, v+725 pp., $5.

*Intermediate Algebra*, L. J. Adams, New York, Henry Holt and Company, 1954. Cloth, v+370 pp., $3.40.

*Introduction to Modern Algebra and Matrix Theory*, Ross A. Beaumont and Richard W. Ball. New York, Rinehart & Company, Inc., 1954. Cloth, v+331 pp., $6.

*Introductory College Mathematics*, Chester George Jaeger and Harold Maile Bacon, New York, Harper and Brothers, 1954. Cloth, v+382 pp., $4.75.

*Introductory College Mathematics*, Adele Leonhardy, New York, John Wiley & Sons, Inc., 1954. Cloth, v+459 pp., $4.90.

*Mathematics for Students of Engineering and Applied Science*, L. B. Benny, New York, Oxford University Press, 1954. Cloth, vi+783 pp., $5.60.

*Mathematics in Agriculture* (Second edition), R. V. McGee, New York, Prentice-Hall, Inc., 1954. Cloth, v+208 pp., $4.

*Methods of Algebraic Geometry, Volume III, Book V: Birational Geometry*, W. V. D. Hodge and D. Pedoe, New York, Cambridge at the University Press, 1954. Cloth, vii+336 pp., $7.50.

*Theory of Functions of a Complex Variable* (Volume I), C. Carathéodory, translated by F. Steinhardt, New York, Chelsea Publishing Company, 1954. Cloth, ii+301 pp.

*A Treatise on Conic Sections Containing an Account of Some of the Most Important Modern Algebraic and Geometric Methods* (Sixth edition), George Salmon, New York, Chelsea Publishing Company, 1954. iv+399 pp., paper, $1.94; cloth, $3.25.

*Understanding College Algebra*, Edwin R. Smith, Samuel Selby, and Murray Kleiman, New York, The Dryden Press, 1954. Cloth, v+573 pp., $3.50.

*Vector and Tensor Analysis*, G. E. Hay, New York, Dover Publications, Inc., 1953. v+193 pp., cloth, $2.75; paper, $1.50.

MISCELLANEOUS

*Concerning the Nature of Things*, Sir William Bragg, New York, Dover Publications, Inc., 1954. v+231 pp., cloth, $2.75; paper, $1.25.

*Essays in Science*, Albert Einstein, New York, Philosophical Library, Inc. Cloth, vii+114 pp., $2.75.

*Foundations of Potential Theory*, Oliver Dimon Kellogg, New York, Dover Publications, 1953. vi+384 pp., cloth, $3.95; paper, $1.90.

*MATHeMAGIC Magic, Puzzles, and Games with Numbers*, Royal Vale Heath, New York, Dover Publications, Inc., 1953. Paper, 126 pp., $1.

*Matter Energy Mechanics*, Jakob Mandelker, New York, Philosophical Library, 1954. Cloth, vii+73 pp., $3.75.

*The Nature of Light and Colour in the Open Air*, M. Minnaert, translation H. M. Kremer-Priest, revision K. E. Brian Jay, New York, Dover Publications, Inc., 1954. v+362 pp., cloth, $3.95; paper, $1.95.

*Henri Poincaré—Critic of Crisis-Reflections on his Universe of Discourse*, Tobias Dantzig, New York, Charles Scribner's Sons, 1954. Cloth, vii+149 pp., $3.

*The Principles of Physical Optics—an Historical and Philosophical Treatment*, Ernst Mach, translated by John S. Anderson and A. F. A. Young, New York, Dover Publications, Inc., 1953. v+324 pp., cloth, $3.50; paper, $1.75.

*Science in Everyday Life*, Ellsworth S. Obourn, Elwood D. Heiss, and Gaylord C. Montgomery, New York, D. Van Nostrand Company, Inc., 1953. Cloth, iii+612 pp., $3.80.

*The Superintendency of Public Schools—An Anxious Profession*, Willard B. Spalding, Cambridge, Harvard University Press, 1954. Cloth, 53 pp., $1.50.

## EQUIPMENT

*C-Thru A-1 Ruler*, C-Thru Ruler Company, 839 Windsor Street, Hartford, Connecticut. Plastic ruler with eight cutout figures, $1\frac{1}{8}'' \times 12''$, $14.40 per gross (list price).

*Slide Rule Kit*, Dr. Kenneth P. Kidd, College of Education, University of Florida, Gainesville, Florida. Kit of cardboard strips and scales with manual for construction, single kit $0.25; ten or more kits $0.20 each.

## PAMPHLETS

*ABC's of Hand Tools*, Educational Relations Section, Department of Public Relations, General Motors, Detroit 2, Michigan. Pamphlet, 49 pp., single copy free.

*Can I Be a Craftsman?* Educational Relations Section, Department of Public Relations, General Motors, Detroit 2, Michigan. Pamphlet, 20 pp., single copy free.

*Facts about Positions in IBM*, IBM Department of Information, 590 Madison Avenue, New York 22, New York. Pamphlet, 31 pp., free.

*Free and Inexpensive Aids for the Teaching of Mathematics, Circular No. 348 (Revised), January, 1954*, Publications Section, Office of Education, Department of Health, Education, and Welfare, Washington 25, D.C. Pamphlet, 7 pp., free.

*Light on the Future*, IBM Department of Information, 590 Madison Avenue, New York 22, New York. Pamphlet, 30 pp., free.

*Manpower Resources in Mathematics*, Superintendent of Documents, Government Printing Office, Washington 25, D.C. Pamphlet, 22 pp., Catalog No. NS 1.2: M 42, $0.20.

*Mathematics A Key to Manpower* (Reprint from *School Life*, November, 1953), Publications Section, Office of Education, Department of Health, Education, and Welfare, Washington 25, D.C. Pamphlet, free.

*Opportunities Unlimited: The Engineer in IBM Laboratories*, IBM Department of Information, 590 Madison Avenue, New York 22, New York. Pamphlet, 32 pp., free.

*Power Goes to Work*, Educational Relations Section, Department of Public Relations, General Motors, Detroit 2, Michigan. Pamphlet, 136 pp., single copy free.

*Professional Opportunities in Mathematics*, 2nd edition, 1954, Professor H. M. Gehman, Mathematical Association of America, University of Buffalo, Buffalo 14, New York. Pamphlet, 24 pp., $0.25 for single copies; $0.10 each for orders of ten or more.

*Selected Bibliography of Current Articles in Mathematics Education, Circular No. 346 (Revised)*, Publications Section, Office of Education, Department of Health, Education, and Welfare, Washington 25, D.C. Pamphlet, 6 pp., free.

*Selected Bibliography of Reference and Enrichment Material for the Teaching of Mathematics, Circular No. 347 (Revised)*, Publications Section, Office of Education, Department of Health, Education, and Welfare, Washington 25, D.C. Pamphlet, 7 pp., free.

*Teaching Materials for Mathematics Classes, Circular No. 399*, Publications Section, Office of Education, Department of Health, Education, and Welfare, Washington 25, D.C. Pamphlet, 36 pp., free.

*1900 to 19XX: A Report on Automobile Engine Progress*, Educational Relations Section, Department of Public Relations, General Motors, Detroit 2, Michigan. Pamphlet, 36 pp., single copy free.

## TESTS

*Blyth Second-Year Algebra Test*, M. Isobel Blyth, World Book Company, Yonkers-on-Hudson 5, New York. Test, $2.80 net per package of 35; machine-scored answer sheets $1.15 net per 35; specimen set $0.35 postpaid.

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## The editor's mail

First of all a word of commendation on the fine article "Goals in Algebra" by Mr. Adkins in the May issue of THE MATHEMATICS TEACHER. I was particularly pleased with his plea for teaching the high-school student *what mathematics is all about*, if indeed we are unable to teach him *all about mathematics*. So relatively few of our students will be going on to pursue mathematics or science as a career, and of those who do, scarcely one sees mathematics as anything but a tool with which to solve practical problems. Why must we withhold the deeper insights until the advanced undergraduate or graduate level? This, of course, presupposes that the teachers themselves can answer that question posed by Mr. Adkins: "What is Mathematics all about?" If they can appreciate the inner richness of the mathematical method, then, it seems to me, they can make it palatable and fascinating for the high-school mind.

(*signed*) JOSEPH G. MURRAY, S.J.
Bellarmine College
Plattsburg, N. Y.

I have one concern regarding the plans for THE MATHEMATICS TEACHER as discussed in "Points and Viewpoints" for January. I refer to the comment " . . . articles on arithmetic will disappear from THE MATHEMATICS TEACHER." I have had many secondary school teachers in my arithmetic methods classes who have become very interested in the ideas of arithmetic and have thereby better understood the mathematics that they were teaching. There is no question but that the orientation of the articles should be changed so they will be more concerned with phases of arithmetic which are especially useful to secondary school teachers. Clamurro's article in the January issue of THE MATHEMATICS TEACHER is an illustration of what I refer to. THE MATHEMATICS TEACHER is one of the most influential professional magazines published today, and some of this strong influence may be lost if the magazine doesn't continue to bring to the secondary school teachers the valuable helps which are available from the elementary school arithmetic program.

(*signed*) ROBERT L. BURCH
Ginn and Company

*A column of unofficial comment.*

# *News for Council members*

*by Marie S. Wilcox, President of The National Council of Teachers of Mathematics.*

Mathematics teachers will welcome a new feature in this journal beginning with this issue. Under the heading, "Your professional dates," (see p. 427) are listed announcements of dates and places of professional meetings. Important among these "professional dates" are the dates of the four meetings of the National Council for the coming year.

The Christmas meeting will be held at the Chase Hotel in St. Louis, December 28 and 29, with some informal activities on December 27. Teachers who plan to attend are urged to arrange hotel reservations at an early date to secure special convention rates. Miss Mary Rogers is program chairman for this meeting, and Dr. Margaret Willdering and Dr. Jesse Osborn are co-chairmen of local arrangements.

The Statler Hotel in Boston, Massachusetts, will be the headquarters for the 1955 annual meeting, April 13 through 16, 1955. Jackson Adkins of Phillips Exeter Academy, Exeter, New Hampshire, will serve as general chairman of arrangements. The president and four vice-presidents of the Council will arrange the program. One of the general meetings will probably be held in the auditorium of the beautiful John Hancock Building. Walking tours as well as bus trips to see historic Boston are being arranged by local committees.

MARIE S. WILCOX

June 27, 1955, will be the date of the one day meeting of the Council at the time of the 1955 NEA convention in Chicago, Illinois.

The 1955 summer meeting will find Council members at Indiana University, Bloomington, Indiana. Housing will be in new university dormitories and meetings are all scheduled for air-conditioned rooms in the nearby Education Building. Dr. Philip Peak made these attractive arrangements and will make other local arrangements for the meeting.

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The National Council was invited to have representatives at the Conference of the National Commission of Teacher Education and Professional Standards held in Albany, New York, this past summer. C. V. Newsom, Ralph A. Beaver, Dorothea C. Hadsell, and Margaret Childers represented the Council.

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The presidents and executive secretaries of groups affiliated with the NEA were invited to meet with the NEA committee concerned with international relations at the time of the NEA meeting in New York last summer. The problem of having a full-time accredited observer at the UN representing public education in this country was presented to this group. Robert H. Reid of the NEA is an accredited observer but maintains his office and spends most of his time in Washington, D.C. Someone representing the NEA and other professional education organizations such as The National Council of Teachers of Mathematics is not always available when an agency or commission of the UN may desire to consult such a person. Neither is it possible under present conditions for groups such as the National Council to be adequately informed as to activities at the UN affecting education.

This meeting resulted in a request to the Recommendations Committee of the NEA to present a resolution to the representative assembly requesting the Executive Committee of the NEA to take the initiative in formulating a plan by which adequate representation could be established and maintained at the UN. The National Council will probably be invited to participate in further discussions concerning this matter.

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"Give me one of those application blanks, too, and I will join the National Council. I should have done it long ago!" This comment was overheard at the registration table at the National Council meeting in New York this past summer.

Many teachers "should have joined long ago" but just didn't have an application blank available.

A Membership Committee with Mary Rogers as chairman has been appointed to co-operate with state chairmen, affiliated groups, and individual members to see that interested teachers do have membership blanks available. The Council now has 10,000 members. This committee has set a goal of 15,000 members by 1956!

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Many members of the Council have probably been concerned about the effect that the publication of *The Arithmetic Teacher* might have on the membership fee and the journal received by members. No change has been made. THE MATHEMATICS TEACHER is still the journal received by members. Other journals may be bought by subscription or individual copy.

However, the Board of Directors of the Council at its meeting last April authorized the appointment of a committee to study membership dues and other charges to members of the Council. Lenore John of the Laboratory School of The University of Chicago is chairman of this committee and will welcome suggestions from members of the Council.

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## Have you read?

FOUCH, ROBERT S. "The Un-Sanity of Mathematics and Its Teaching," *ETC: A Review of General Semantics*, Winter 1954, pp. 113–121.

You cannot afford to miss this article on the semantic sins of mathematics teachers. The author makes a clear statement as to how these sins will affect the future generations who have need of mathematics. For example, how does number differ from numeral? What is the difference between fractional and fraction? Are $2+5$, and 7 equal quantities or the same quantity with two different names? If $a=b$, are they different or two names for the same thing? You will be surprised with his comment on signed numbers. In a triangle, what is meant by "$AC=BC$," when in the same triangle "$AC \perp BC$"? Does "$AC$" mean the same in both situations? What all could "$AC$" mean? You will also be interested in comments on infinity, parallels meeting at infinity and $f(x)=e^x \sin x$. "There is identification, reflection, major play on words and symbols, excessive abbreviation, over generalization, and poor contact with reality." Whether you subscribe to this basic philosophy or not we can well take some of it to heart.—*Philip Peak, Indiana University, Bloomington, Indiana.*

# NCTM

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

## *Minutes of the Annual Business Meeting*

The National Council of Teachers of Mathematics
Gibson-Sheraton Hotel, Cincinnati, Ohio
Saturday April 24, 1954

The meeting was called to order by the President, John R. Mayor, who had the attendance checked and a quorum was declared present.

The reading of the minutes of the April 11, 1953, meeting, in Atlantic City, was called for.

Motion by M. F. Rosskopf and seconded by Ben Sueltz: That, the minutes be approved as printed in the October, 1953, issue of THE MATHEMATICS TEACHER, and to suspend the reading of the minutes.

Motion carried.

The first item of business was the Proposed Revision of the By-Laws. The report was made by Lynwood Wren, a member of the committee, who read the following:

> Since plans are under way for the National Council to publish a journal for teachers of arithmetic and, also, one for high school students, a few changes in the By-Laws of the Council should be made. A committee has studied this matter and recommends the changes shown below. A complete copy of the present By-Laws will be found in THE MATHEMATICS TEACHER for February, 1951.

(Copies were in the hands of all present.)

The following action was taken:

Article II, #2.

> The Board of Directors may establish various types of membership in the Council and determine the fee for each, and designate the publications included. THE MATHEMATICS TEACHER shall be the official journal.

L. Wren moved the acceptance of the revision of Article II, #2. B. Sueltz seconded the motion.

Motion carried.

Article III, #1.

> Officers. The officers of the Council shall be a president, four vice-presidents, a recording secretary, and executive secretary, and an editor for each Journal published by the Council.

L. Wren moved the proposed changes be accepted in Article III, #1. J. Zant seconded the motion.

Motion carried.

Article III, part of #3.

> The Executive Secretary, Recording Secretary, and Editors shall be appointed by the Board of Directors. The Executive Secretary shall be appointed for a term of three years; he may be re-appointed. The Editors shall be appointed for a term of three years; they may be re-appointed but shall not serve for more than two consecutive terms. The Recording Secretary shall be appointed for a term of one year; he may be re-appointed.

L. Wren moved the adoption of the proposed change in Article III, part of #3. Z. L. Loflin seconded the motion.

Motion carried.

Article III, #5.

> (5) Editors. Each Editor shall be responsible for his journal in all phases except that of business management.

L. Wren moved the acceptance of the revision in Article III, #5. C. Christofferson seconded the motion.

Motion carried.

Article III, #9. Duties of Committees.

> (5). The Committee on Supplementary Publications shall supervise the editing and publishing of all publications except Year Books, Journals, and those for which other provisions are made by the Board.

L. Wren moved the acceptance of the revision in Article III, #9. L. Hall seconded the motion.

Motion carried.

Article III, #10. Editorial Boards.

> Each journal shall have an Editorial Board consisting of an Editor and such associate editors as designated by the Board of Directors. The associate editors shall be ap-

pointed for a term of three years by the Board of Directors on recommendation of the Editor for that particular journal. They shall be eligible for re-appointment but shall not serve for more than two consecutive terms.

L. Wren moved the acceptance of revision in Article III, #10. V. Schult suggested that the words "of Directors" be inserted at the end of the first sentence. The suggestion was accepted by L. Wren and those present. I. M. Bernhard seconded the motion.

Motion carried.

A brief discussion on the question of membership with reference to the new publications, namely, *The Arithmetic Teacher*, brought a suggestion from Mr. Frey that perhaps the four issues of *The Arithmetic Teacher* and the four reports of official announcements might serve the same as eight issues of THE MATHEMATICS TEACHER and allow membership, if the same cost was involved.

John Mayor remarked that the new publications did bring up problems on membership and the Board of Directors would be glad to hear opinions.

The next item of business was a resolution presented by Agnes Herbert, chairman of the Committee on Relations with NEA.

*Resolution:*

*Whereas*, the National Council of Teachers of Mathematics is a department of the National Education Association, and as such receives benefits,

*Be it resolved*, that the National Council of Teachers of Mathematics include in its program, in so far as possible, the CAP (Centennial Action Program) of NEA, further encourage membership, particularly Life Membership in NEA, in support of the NEA Building Fund.

A. Herbert moved the adoption of the resolution. A. Archer seconded the motion. A brief discussion of the resolution followed.

Motion carried.

Veryl Schult, Chairman on Resolutions concerning the NASSP read the following: Resolved that:

1. The National Council of Teachers of Mathematics express its appreciation of the opportunity given it by the National Association of Secondary School Principals to prepare the May issue of the Bulletin of NASSP, and thus to present the picture of mathematics today to such a wide and influential audience.
2. Since there is such a critical shortage of scientific manpower and of teachers trained in mathematics, the NCTM recommends that every high school provide means for a four-year sequence of study in mathematics for all students having the ability and the need for it. The NCTM urges the serious consideration of this recommendation by the National Association of Secondary School Principals.

*Clifford Bell*
*Howard Fehr*
*Henry Syer*
*Veryl Schult, Chairman, Committee on Resolutions concerning the NASSP*

After reading the resolution, Veryl Schult moved that the resolution be accepted. Motion seconded by P. Jones. Discussion followed. Bruce Meserve suggested Resolution #2 be revised so as to take in more professional groups in the NEA. Phillip Jones suggested that the Resolutions be sent to Affiliated Groups and other State Groups. Harry Charlesworth suggested copies be sent to the State Departments.

Motion by Mr. Christofferson, seconded by L. Wren: That the words, "National Association of Secondary School Principals," be deleted and the Resolutions be sent by the Executive Secretary to all professional organizations which seem appropriate.

Motion carried.

Ida May Bernhard presented the following Resolution and moved its adoption:

The President, Vice-Presidents, additional members of the Board of Directors, all other officers and members-at-large of the NCTM, wish to express their sincere thanks and appreciation to the Ohio Council of Teachers of Mathematics, the Mathematic Club of Greater Cincinnati, the Local Committees, the WKRC, TV, the Cincinnati *Times Star*, the Cincinnati *Post*, the Cincinnati *Inquirer*, Sheraton-Gibson Hotel, and all others who shared in planning this meeting in the City of Cincinnati. The friendliness, hospitality and co-operation show that your efforts have been limitless in seeking to make this a truly great and memorable meeting.

I. M. Bernhard moved the adoption of the resolution. Lynwood Wren seconded the motion.

Motion carried.

Motion by Lynwood Wren, seconded by Veryl Schult: That, the members of the NCTM express their appreciation to John Mayor for very fine leadership during his term of office.

Motion carried.

M. G. Ahrendt reported briefly for the Washington Office. He stated that the Council had finally reached its goal of 10,000 members, a count on April 21st showing 10,081 plates and orders in the files.

He stated that the Council is being operated on a sound financial basis. Improved accounting procedures have enabled us to evaluate accurately our total assets and liabilities. Our most recent financial statement on March 31, 1954, showed cash resources (in round numbers) of $39,000, and our inventory of publications on December 1, 1953, was valued at $13,000.

The subscription lists are building up satisfactorily on the two new journals. Enough subscriptions for one journal were received during the first six weeks to make it self-supporting. The other will be self-supporting by the time the third issue appears if the subscription list continues to build up at the present rate.

A number of new publications have become available from the office during the past year. These include the revised edition of the "Guidance Pamphlet," the Twenty-Second Yearbook, a special issue of the "NASSP Bulletin," and several reprints from THE MATHEMATICS TEACHER. Other small publications are on the way.

Mary Rogers, Chairman of the Affiliated Groups reported for the Affiliated Groups. There are five regional representatives and this group had their Fifth Delegate Assembly at which many important items were discussed. The points of major interest were:

1. The Handbook—may be ready for distribution at the Sixth Delegate Assembly
2. The *News Letter*
3. An enlarged Agenda Planning Committee
4. Program participation sponsored by State Councils
5. Financial statement
6. Question of size—"How large must a group be to become an Affiliated Group?" A membership of fifty was suggested, but no conclusion drawn.

Agnes Herbert gave the following data concerning future places of meeting:

Summer, 1954, Seattle, Washington, August 22–25
Christmas, 1954, St. Louis, Missouri
Spring, 1955, Boston, Massachusetts
Summer, 1955, May be at the University of Indiana, Bloomington, Indiana
Christmas, 1955, Washington, D.C.
Spring, 1956, Milwaukee, Wisconsin

Ben Sueltz, Editor of *The Arithmetic Teacher* made the following report on the new publication:

As of April 1st, we had 1860 subscriptions. On April 20th, more than 2000. It is hoped that this will be increased to 4000 during the next year.

The editor of *The Arithmetic Teacher* met with the advisory committee of the Council in November and was charged with the development of a quarterly journal. He started from "scratch" and with the aid of a representative from the Banta Publishing Company proceeded to design the magazine, its cover, layout, type faces, sizes, etc. But any good journal depends upon many people and not upon the editors alone.

*The Arithmetic Teacher* aims for a style of writing that is simple and direct. Because the number of pages is limited to thirty-two, each issue usually cannot have more than three or four main presentations. By reducing the space between lines the equivalent of an additional four pages is made available. Does anyone object seriously to the page makeup of the journal? Since there is objection, the leading will be changed.

*Finances.* To date, the editor has spent $91.89 for office supplies and experses. More supplies will be needed and purchased. However, the expenses of the editor will remain well within the budget. The aim of the new quarterly journal is one of service, a service to the boys and girls of this country through their teachers and supervisory officers.

Harold Larson, Editor of *The Mathematics Student Journal* made the following report:

The latest count showed a total of 30,000 subscriptions to *The Mathematics Student Journal.* In fact, the journal was self-supporting before the first number was issued. It is hoped that the subscriptions will continue to increase at the present rate so that the magazine can be expanded in the near future to eight pages without an increase in price. An appeal was made for more contributed items from students and teachers. A large backlog of material is necessary to produce an interesting, instructive, and well-balanced issue of the *Journal.*

Henry Van Engen, Editor of THE MATHEMATICS TEACHER, gave a brief report. He expressed appreciation for the help and co-operation given during the past year in reading manuscripts and making suggestions.

Donovan Johnson asked that the NCTM express appreciation to the following persons for the fine work they have done in the NCTM publication of journals:

Henry Van Engen
Ben Sueltz
Harold D. Larson

Henry Syer made a few remarks in reference to the Committee of Publications of Current Interest. He said the committee (1) needs manuscripts; (2) wished to have some suggested titles for manuscripts; and (3) were desirous of having reactions to the publications.

Lynwood Wren reported on the Yearbooks. He told of a long range program, with editors L. Wren for the twenty-third and Phillip Jones for the twenty-fourth, already approved by the Board of Directors.

Vera Sanford gave the following report on the Recognition of Council Members who have retired:

The Committee suggests that:

1. An honorary or emeritus membership in the NCTM be established for those who retire from teaching with a record of at least twenty-five years of continuous membership in the Council at the time of their retirement. These honorary members should not pay dues. They should have a vote in Council elections and

should be eligible for service on committees.

2. The Board of Directors should set up a committee charged with the duty of recommending, to the Board, those who qualify for honorary membership as stated above, and of making recommendations to the Board of those whose service to the Council has been outstanding and who should be given additional appropriate recognition. In compiling a list of those to receive honorary memberships, it is suggested that the committee enlist the services of the Executive Secretary and of the representatives of the Affiliated Groups.

3. It is also suggested that a certificate of honorary membership be prepared and that the list of those appointed to this membership be published in THE MATHEMATICS TEACHER at the time of their appointment.

Respectfully submitted,
*James Zant*
*Ida May Bernhard*
*Mamie Ausrbach*
*Glenn Ayre*
*Vera Sanford, Chairman*

No action was taken on this report.

Agnes Herbert made the following report for the Committee on Nominations and Elections: The By-Laws pertaining to nominations and elections were read. Members were asked to submit names of candidates with a statement of qualifications by August 1, 1954, to any member of the committee, or to the Chairman, Agnes Herbert, 806 East North Avenue, Baltimore 2, Maryland.

JACKSON ADKINS, New Hampshire
CLIFFORD BELL, California
HARRY CHARLESWORTH, Colorado
MARY POTTER, Wisconsin
MAURICE HARTUNG, Illinois
HOUSTON KARNES, Louisiana
C. V. NEWSOM, New York
MARY ROGERS, New Jersey
AGNES HERBERT, Maryland, *Chairman*

John Mayor then introduced the new officers:

President—MARIE WILCOX
Vice-President—Senior High—
VERNON PRICE
Vice-President—Elementary—
CHARLOTTE JUNGE
Members of the Board—
WILLIAM GAGER CLIFFORD BELL
CATHERINE LYONS

He expressed his thanks and appreciation to the 10,081 members for their loyal support during his term as President and then turned the gavel over to the new President, Marie Wilcox.

Marie Wilcox accepted the gavel from John Mayor with these remarks:

"Thank you, Dr. Mayor. I receive this gavel, as you probably know better than anyone else at this moment, with mixed emotions. I have been closely associated with the work of the Council in the past few years and hence realize the great responsibility which is associated with the office of president. However, no one would permit his name to be placed in nomination for the office of president if he had not been closely enough associated with the work of the Council to know that there is a very large number of members who are willing to assist with this work.

"I should like to take this opportunity to thank the members of the Council for the vote of confidence which they have given me. I should like to say that I consider the office of president as an executive in nature and shall, therefore, endeavor to carry on the affairs of the Council in the manner directed by the Board of Directors, in accordance with the wishes of the membership, and, I hope, in a manner which will always be in the best interests of the Council."

The meeting was then declared adjourned.
Respectfully submitted,
*Agnes Herbert, Recording Secretary*

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## Registrations at the Fourteenth Christmas Meeting

*The National Council of Teachers of Mathematics, Los Angeles, California, December 28–30, 1953*

| | |
|---|---|
| Arizona | 10 |
| California | 255 |
| Colorado | 8 |
| District of Columbia | 1 |
| Illinois | 7 |
| Indiana | 2 |
| Iowa | 2 |
| Kansas | 4 |
| Michigan | 1 |
| Minnesota | 2 |
| Nebraska | 2 |
| New Mexico | 9 |
| New York | 2 |
| Ohio | 2 |
| Oregon | 5 |
| Texas | 1 |
| Utah | 1 |
| Washington | 5 |
| Wisconsin | 5 |
| Wyoming | 1 |
| Foreign | 2 |
| Total | 327 |

# *Minutes of Fifth Delegate Assembly*

## The National Council of Teachers of Mathematics
## Hotel Sheraton-Gibson, Cincinnati, Ohio
## April 22, 1954

*Edited by Mary C. Rogers, Chairman Affiliated Groups 1952–1954, Roosevelt Junior High School, Westfield, New Jersey*

### First session of the Assembly

The Fifth Delegate Assembly of The National Council of Teachers of Mathematics was called to order by the Chairman, Mary C. Rogers.

Miss Rogers welcomed the fifty-four delegates, eight alternates, and fourteen special guests in attendance at the Assembly session. She introduced Dr. Daniel Lloyd, Chairman, and other members of the Agenda Planning Committee.

The five Regional Representatives were presented and each in turn introduced the delegates and alternates from his area. The Regional Representatives and the states included in their areas are as follows:

Northeastern Area—Jackson B. Adkins, Phillips Exeter Academy, Exeter, New Hampshire, [District of Columbia, New England States, Delaware, New Jersey, New York, Pennsylvania].

Southeastern Area—William A. Gager, University of Florida, Gainesville, Florida [Alabama, Florida, Georgia, Kentucky, Maryland, North Carolina, South Carolina, Tennessee, Virginia, West Virginia].

North Central Area—Irene Sauble, Detroit Public Schools, Detroit, Michigan [Illinois, Indiana, Iowa, Michigan, Minnesota, Ohio, Wisconsin].

Northwestern Area—Elizabeth Roudebush, Seattle Public Schools, Seattle, Washington [Idaho, Kansas, Missouri, Montana, Nebraska, North Dakota, Oregon, South Dakota, Washington, Wyoming].

Southwestern Area—Ida May Bernhard, Texas Education Agency, Austin, Texas [Arizona, Arkansas, California, Colorado, Nevada, New Mexico, Oklahoma, Texas, Utah, Louisiana, Mississippi].

Miss Rogers expressed the enthusiasm of the Board of Directors in the excellent work being done by the Affiliated Groups. She complimented the Regional Representatives on their leadership and their services to the groups in their areas.

Mr. Harry Charlesworth and Dr. John R. Mayor were presented in respective order as the first and second chairmen of the Committee on Affiliated Groups. Dr. Mayor, as President of the National Council, welcomed the delegates and expressed his sincere interest in the work of the Affiliated Groups. He expressed his appreciation for the great contribution which the Groups have made to the growth and success of the National Council during the past five years. He requested that the organizations continue to contribute ideas for the improvement and extension of services of the Council.

Miss Rogers reported that at present there are only four states in which there is no affiliated group. Since April, 1953, certificates of affiliation have been awarded to four new groups:

1. Michigan Council of Teachers of Mathematics
2. Missouri Council of Teachers of Mathematics
3. Association of Mathematics Teachers of New York
4. Puget Sound Council of Teachers of Mathematics

Miss Elizabeth Roudebush issued an invitation to attend the summer meeting of the National Council at the University of Washington, Seattle, August 22–25. She asked each delegate to take five copies of the program for members of his Affiliated Group.

Miss Rogers requested that the Affiliated Groups give detailed information about their organization and its activities on the renewal of affiliation forms submitted each year. These renewal forms reveal some very interesting and encouraging information; for example, the total reported local membership for 1953–54 shows an increase of more than 24 per cent over last year; forty-one of the fifty-three groups reporting memberships in local organizations show a gain over last year's records. The number of meetings held per year per organization varies from one to nine, the average being three. Some of the Affiliated Groups sponsor excellent newsletters and other publications. Miss Rogers expressed her gratitude for these local news letters and their mention of National Council and its Activities. She stressed the need for an early return of renewal reports and the need for receipt of a revised list of the officers of the state and local organizations early in the fall. The editor of the *News Letter* needs to know to whom to write for news and the Regional Representatives need to know to whom to address information of interest to Affiliated Groups in their areas.

The attention of the delegates was called to the fact that information on affiliation with Na-

tional Council is contained in the November, 1950, issue of THE MATHEMATICS TEACHER. Reprints of the article containing this information are available upon request. If a group wants to affiliate, it should contact the Regional Representative from its area for all forms and information.

Miss Rogers reported the good news that the Board of Directors has voted $300.00 to the work of Affiliated Groups in 1954–55 as compared with $100.00 of this year.

Mr. M. H. Ahrendt, Executive Secretary of National Council, gave the following report from the Washington Office:

1. On April 12 the membership of National Council was 10,081.

2. The financial condition of the Council is sound.

3. The two new publications, *The Arithmetic Teacher* and *The Mathematics Student Journal*, have been received with interest and enthusiasm. One of the publications is already paying for itself and the other should be as well by October.

4. The "Guidance Pamphlet in Mathematics for High School Students" has been revised recently. The Twenty-Second Yearbook, entitled *Emerging Practices in Mathematics Education*, is off the press and on sale at $3.50 to members of N.C.T.M. and $4.00 to nonmembers.

Mr. Ahrendt requested that the group give some attention to ways of strengthening bonds between the State Representatives and the Affiliated Groups. Dr. Myron Rosskopf asked how the State Representatives are chosen and how long they serve. Mr. Ahrendt replied that there is no set way of choosing the State Representatives but that the Board of Directors recommend that they be elected or appointed within the Affiliated Group representing an entire state. He said that the State Representatives in the past have served as long as they were willing to work for the Council in that capacity.

The discussion which followed indicated the feeling in some states to be that the lack of uniformity in the choice of State Representatives is not good. Dr. Daniel Lloyd made a motion that a committee from the Assembly be appointed to study this problem and to report its conclusions to the Board of Directors at an early date. The motion was seconded by Dr. Milton Beckman and carried. The following had shown interest in the problem and agreed to serve on the committee: Dr. Myron Rosskopf, Mr. Robert Fouch, and Mr. Russell Dineen.

Dr. H. Van Engen, Editor of THE MATHEMATICS TEACHER, reported that 11,000 copies of the official publication of National Council are now being printed each month. He distributed a questionnaire with a request for expression of opinion as to the kind of articles appearing in the magazine. These questionnaires were returned to Dr. Van Engen at the end of the Assembly.

Dr. Van Engen explained a plan to add a section in THE MATHEMATICS TEACHER with the title "Dates to Remember" or something similar. In this section would be dates of meetings of National Council and of local and state groups affiliated with the Council. For such a section to be possible, the Affiliated Groups must send the information to the editor at least two months before printing. Mrs. Ben F. Pillow suggested that information of workshops and institutes be grouped together in the magazine, that this information include date, place, name of sponsoring organization, and name and address of person to whom to write for further information. The suggestion was noted by the editor.

Mr. Robert Fouch asked Dr. Van Engen for some comment as to the manuscripts for THE MATHEMATICS TEACHER. The reply was that the editorial board is particularly in need of manuscripts on seventh and eighth grade mathematics, also articles on interpretation of mathematics to high-school teachers, and articles giving different points of view in teaching specific concepts and ideas.

Dr. Ben Sueltz, Editor of *The Arithmetic Teacher*, reported that at present there are 2,000 subscribers to the magazine. It is written especially for elementary school teachers, and he is desirous that the style of writing be simple and direct. He wants articles on emerging practices in the teaching of arithmetic. Dr. Sueltz would like the magazine to offer an opportunity for the sharing of experiences among teachers.

The delegates expressed comments from elementary school teachers for the need of increase in space between the lines of print so that reading of the magazine might be easier.

The difficulty in getting elementary school teachers both to join National Council and to subscribe to *The Arithmetic Teacher*, was discussed. The delegates suggested that the Board of Directors work out some plan whereby membership in National Council might entitle one to the choice of either THE MATHEMATICS TEACHER or *The Arithmetic Teacher*.

Mr. H. D. Larsen, Editor of *The Mathematics Student Journal*, told the delegates that on April 12 there were 1,888 group subscriptions, or a total of about 28,500 copies. The publication dates set for the *Journal* are the 15th of February, April, October, and December. The February issue, being the first, was mailed to all members of National Council and to each high school in the United States.

Mr. Larsen stated that the backlog of material is growing satisfactorily; however, he expressed the desire for a more varied type of manuscript. He requested the delegates to spread the news that the editors would like material from teachers and students. Much interest has been shown in the "Problem Page" which is conducted by Mr. Frank Allen, Associate Editor. For those students mailing in correct solutions to the problems a progressive plan of awards might be adopted; that is, certificate, button, pin, medal, etc.

The initial success of *The Mathematics Student Journal* has been great. It is hoped that the number of subscriptions will soon be large enough to warrant expanding the magazine to eight pages without increasing the subscription price.

Dr. F. G. Lankford, Jr., Chairman of Yearbook Planning Committee, presented the following recommendations made by the committee to the Board of Directors:

1. Five Yearbooks be planned now in order to accumulate a backlog of manuscripts in preparation.

The Committee suggested the following titles for these Yearbooks:

a. Interpretations of Modern Mathematics for High School Teachers
b. The Curriculum in Mathematics for Grades 1–12
c. The Education of Mathematics Teachers
d. The Evaluation of Learning in Mathematics
e. In-Service Education for Mathematics Teachers

The Board has approved the beginning of work on topics lettered "a," "d," and "e" and has discussed possible editors.

2. The selection of a co-ordination committee for production of Yearbooks.

Dr. Mayor has been chosen the chairman of this committee which will be composed of the editors of the three Yearbooks in preparation.

The National Council of Teachers of Mathematics has prepared the May issue of the *Bulletin* of the National Association of Secondary School Principals. Mr. John A. Brown, Chairman of the Editorial Committee for the mathematics number of the *Bulletin* reported that the theme of this number was "Meeting Increasing Needs for Mathematics." The purpose is to give to high-school principals practical and helpful suggestions as to ways of providing a program which will more nearly meet the increasing mathematical needs of all pupils.

The contributors of articles are members of the National Council from all sections of the country and representing all levels of instruction.

The May issue of NASSP *Bulletin* may be purchased for 75 cents by members of NCTM and for one dollar by non-members.

Dr. Henry Syer, Chairman of the Committee on Publications of Current Interest, stated the purpose of his committee to be the editing and publishing of small leaflets, booklets, and pamphlets of interest to teachers and pupils of mathematics. There are four kinds of publications which the members of the committee hope to produce:

1. Enrichment material for students.

2. "How To" Series—leaflets giving practical classroom suggestions on some of the techniques of teaching, such as discussion, library work, films and filmstrips, field trips, and bulletin boards.

3. Service publications—bibliographies of various kinds, score sheets for textbooks, or similar aids for the busy teacher.

4. Higher mathematics for teachers—professional reading in the field of mathematics to broaden backgrounds and keep teachers up to date.

Dr. Syer reported that at present there are six manuscripts in the Washington offices ready to be published. He asked for the help of members of the Affiliated Groups in locating good manuscripts, in recommending the types of publications which they would find most useful, and in promoting the booklets when they are published.

Professor W. W. Rankin, Chairman of the Committee on Conferences and Workshops, expressed his great pleasure in the wide acceptance of the idea of institutes. More and more institutes are being set up each year, and the enthusiasm of the teachers in the programs is most gratifying.

Dr. H. Glenn Ayre, Editor of the Affiliated Groups Handbook now in preparation, introduced the members of his committee and spoke of the large amount of work they have accomplished thus far. Dr. Ayre presented to each person a tentative report on the Handbook; he requested the delegates to read the material carefully and to offer their suggestions for improvement.

The tentative outline of the Handbook is as follows:

1. Introduction: The Meaning of Affiliation
2. How to Start a Mathematics Organization
3. Organization
   a. General Organization
   b. Constitution
   c. Selection and Training of Officers
   d. Committees
4. Finances
5. Membership
6. Meetings, Programs, Tournaments, Contests, Exhibits
7. Publications
8. Workshops, Institutes, etc.
9. Procedures Leading to Affiliation—Advantages—Responsibilities
10. Contribution to National Council
11. Cooperation with Other Education Agencies
12. Public Relations

Dr. Rosskopf made the suggestion that a topic by the title, "Responsibilities of National Council to Affiliated Groups," be added to the Handbook.

Dr. Glenn Ayre, as Editor of the *News Letter* of Affiliated Groups, expressed the need of material or news for the May issue. Mrs. Mabel Milldollar suggested that the *News Letter* for May include enthusiastic reactions to the entire convention as expressed by individual delegates. Every local group has the responsibility to send news of its organization for publication.

The report was made that 380 copies of the *News Letter* are mailed, these going to the officers

of the Affiliated Groups. Discussion followed as to possible ways of getting the *News Letter* to reach more people. The following suggestions were made:

1. Send copies of the *News Letter* for distribution when there is a meeting of an Affiliated Group.

2. From officers of Affiliated Group get names of others interested in receiving the *News Letter*.

3. State Representatives and Regional Representatives send in names of key people of the area who might be interested in *News Letter* and who might be influential in forming new Affiliated Groups.

Miss Rogers mentioned the possibility of putting the News Articles of Affiliated Groups in THE MATHEMATICS TEACHER rather than in the separate *News Letter*. By this means more teachers would have a chance to see the news of local groups.

Because of lack of time discussion of this possibility was postponed until the second session of the Assembly.

### Second session of the Assembly

The Second session of the Fifth Delegate Assembly was a dinner meeting held in the Rockwood Room of Hotel Sinton. Miss Mary Rogers presided.

The suggestion made at the first session of having THE MATHEMATICS TEACHER add a department called "Affiliated Group News Department" instead of the separate *News Letter* was discussed. Various opinions on the matter were expressed by delegates. Dr. Ayre asked for a show of hands to see who wanted to incorporate the news items on Affiliated Groups in THE MATHEMATICS TEACHER. Such action showed the majority of the delegates to be in favor of trying the plan.

Mrs. Marie Wilcox was introduced to the group as the new president of National Council.

Miss Rogers presented Miss Agnes Herbert, Chairman of the Committee on Relations with N.E.A., who introduced Dr. Karl H. Berns, Assistant Secretary of N.E.A. in charge of Field Operations. Dr. Berns brought greetings from N.E.A., the parent organization of N.C.T.M. He spoke on the problems and opportunities of N.E.A., with emphasis on the need for an N.E.A. building as the symbol of education in our nation's capital.

Dr. Kenneth Brown, Specialist for Mathematics, U.S. Office of Education, stressed the fact that publicity for National Council can best reach the Affiliated Groups through the news taken back by the delegates.

Mrs. Tolson entertained the group with puzzles taken from a booklet, "Know Your Neighbor," prepared and presented by The Benjamin Banneker Mathematics Club, Washington, D.C.

Upon motion the meeting was adjourned.

Respectfully submitted,
*Mabel Love Baker*
*Charlotte Carlton*
*Secretaries of Delegate Assembly*

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## Student memberships

A student membership in The National Council of Teachers of Mathematics is available for the special price of $1.50 to any student who has never taught. Orders for student memberships must be submitted by or certified by a staff member of the school. The student membership includes a subscription to THE MATHEMATICS TEACHER and all the usual membership privileges except the right to vote and hold office.

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## Revised edition of the Guidance Pamphlet

Readers of THE MATHEMATICS TEACHER will be interested to know that the *Guidance Pamphlet in Mathematics for High School Students* has been revised and enlarged with the help of numerous experts and specialists. Information has been included about new occupational fields for mathematically trained persons. Up-to-date data and statistics have been secured. In view of the increasing importance of mathematics in the modern world, all students, parents, teachers, and counselors should read this pamphlet. The price is $0.25 each, with discounts on quantity orders.

# *Notes from the Washington office*

*by M. H. Ahrendt, Executive Secretary,*
*The National Council of Teachers of Mathematics, Washington, D.C.*

## Membership report

What does one say when he barely succeeds in some attempt? "Whew?" "Thank goodness?" "A miss is as good as a mile?" "I'll do better next time?" It all depends on your viewpoint, of course, and on how close the project was to your heart.

Thus readers of THE MATHEMATICS TEACHER will have varying reactions to the attendance report below. Several years ago we set a goal of 10,000 members. How close we came to failing during the past school year is graphically told by the figures below.

But the goal we did reach is not our final one. It is merely a milestone. If every member of the Council would actively work on membership, we could have 11,000 members enrolled by May, 1955. What will you do to help?

### Memberships in The National Council of Teachers of Mathematics

*May 3, 1954*

| | Individual | Institutional | Total |
|---|---|---|---|
| Alabama | 82 | 31 | 113 |
| Arizona | 26 | 14 | 40 |
| Arkansas | 96 | 18 | 114 |
| California | 373 | 213 | 586 |
| Colorado | 99 | 21 | 120 |
| Connecticut | 114 | 38 | 152 |
| Delaware | 40 | 7 | 47 |
| District of Columbia | 107 | 16 | 123 |
| Florida | 189 | 45 | 234 |
| Georgia | 93 | 31 | 124 |
| Idaho | 8 | 4 | 12 |
| Illinois | 681 | 111 | 792 |
| Indiana | 311 | 50 | 361 |
| Iowa | 149 | 48 | 197 |
| Kansas | 179 | 28 | 207 |
| Kentucky | 56 | 26 | 82 |
| Louisiana | 157 | 27 | 184 |
| Maine | 31 | 15 | 46 |
| Maryland | 160 | 30 | 190 |
| Massachusetts | 241 | 52 | 293 |
| Michigan | 311 | 89 | 400 |
| Minnesota | 198 | 69 | 267 |
| Mississippi | 60 | 22 | 82 |
| Missouri | 165 | 55 | 220 |
| Montana | 31 | 9 | 40 |
| Nebraska | 92 | 18 | 110 |
| Nevada | 4 | 5 | 9 |
| New Hampshire | 35 | 8 | 43 |
| New Jersey | 295 | 79 | 374 |
| New Mexico | 39 | 20 | 59 |
| New York | 583 | 204 | 787 |
| North Carolina | 130 | 40 | 170 |
| North Dakota | 18 | 11 | 29 |
| Ohio | 405 | 88 | 493 |
| Oklahoma | 110 | 48 | 158 |
| Oregon | 65 | 24 | 89 |
| Pennsylvania | 442 | 171 | 613 |
| Rhode Island | 35 | 12 | 47 |
| South Carolina | 52 | 39 | 91 |
| South Dakota | 16 | 8 | 24 |
| Tennessee | 124 | 40 | 164 |
| Texas | 320 | 128 | 448 |
| Utah | 20 | 12 | 32 |
| Vermont | 20 | 7 | 27 |
| Virginia | 188 | 50 | 238 |
| Washington | 117 | 35 | 152 |
| West Virginia | 95 | 10 | 105 |
| Wisconsin | 226 | 54 | 280 |
| Wyoming | 18 | 6 | 24 |
| TOTALS | 7406 | 2186 | 9592 |
| U. S. Possessions | 35 | 14 | 49 |
| Canada | 79 | 55 | 134 |
| Foreign | 110 | 116 | 226 |
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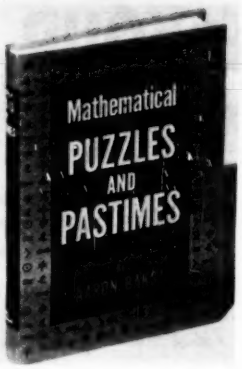


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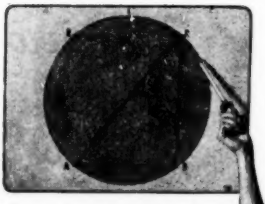

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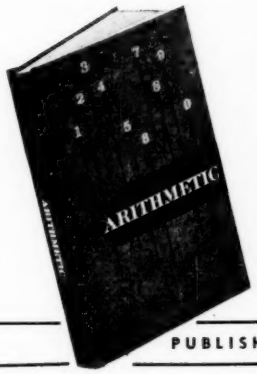


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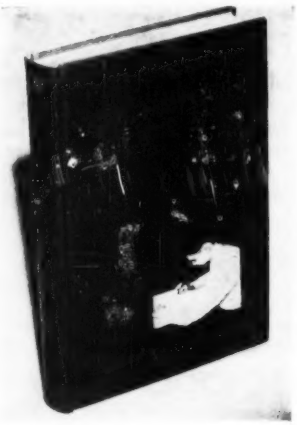

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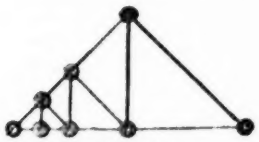
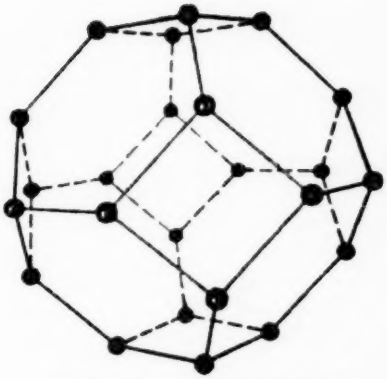
**A FASCINATING NEW TEACHING AID**

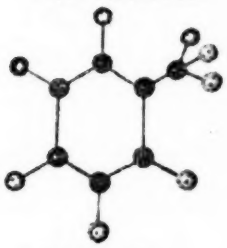


GEOMETRICALLY PROPORTIONED

PYRAMID



TRUNCATED OCTAHEDRON



TOLUENE

SCIENTIFIC MAKIT is the answer to the growing demand for a construction set designed to build geometrical figures, molecular models and other models related to science.

Packed in metal box.

**CONTENTS:**

| | |
|---|---|
| Balls (six colors) | 48 |
| Wheels (3 colors) | 16 |
| Rods (six lengths) | 192 |
| Accessory parts | 32 |
| Total | 288 |
| Price (prepaid) | $5.00 |
| West of Denver | $5.50 |

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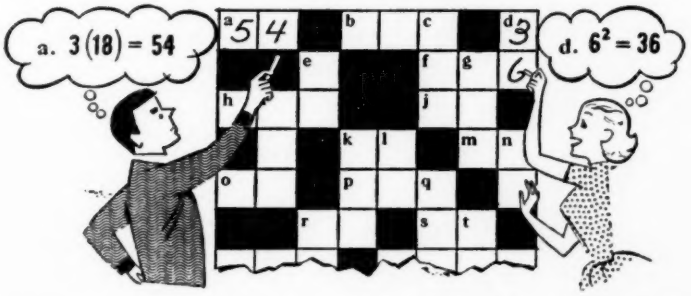


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